ANALYTIC HIERARCHY PROCESS BASED ON THE MAGNITUDE OF Z-NUMBERS

ABSTRACT

The analytic hierarchy process (AHP) is a powerful multi-criteria and multi-alternative decision-making model which helps decision makers in giving preferences using pairwise comparison matrices. The development of AHP using fuzzy numbers got attention from many researchers due to the capability of fuzzy numbers in handling vagueness and uncertainty. The integration of AHP with fuzzy Z-numbers has improved the model since the reliability of decision makers is considered, in which the judgement is followed by the degree of certainty or sureness. Most of the existing decision-making models based on Z-numbers transform the Z-numbers into regular fuzzy numbers by integrating the reliability parts into the restriction parts which has caused a great loss of information. Hence, this research develops the AHP based on the magnitude of Z-numbers, in which the magnitude is used to represent the criteria weights. A numerical example of criteria ranking for the prioritization of public services for digitalization is implemented to illustrate the proposed AHP model.

Keywords: AHP, magnitude, Z-numbers, criteria ranking.

1. Introduction

Many multi-criteria decision-making (MCDM) methods have been developed to help decision makers in selecting the best alternatives when there are various attributes. The AHP is one of the powerful methods which was proposed by Saaty (1980) in which the pairwise comparison matrix was used to obtain the evaluation by decision makers. The AHP has been studied extensively due to the fact it is simple, easy to use and flexible. It has been implemented to solve decision-making problem with many criteria in various fields such as education, management, engineering, manufacturing and sports.

2. Literature Review

Since the emergence of the theory of Z-numbers by Zadeh (2011), many works have been developed to perform the calculation on Z-numbers. Most of them converted Z-numbers into regular fuzzy number for simplicity, which was initiated by Kang et al. (2012). However, this conversion process has led to the information loss. Aliev et al. (2015, 2016) proposed of performing arithmetic operations directly on Z-numbers without going through the conversion process. However, their proposed operations involved linear programming which has caused the computational complexity just to solve simple problems (Abdullahi et al., 2020). Hence, a new method of ranking Z-numbers was proposed by Farzam et al. (2021) considering the magnitudes of the restriction (A) and reliability (R) components as follows:

$$\mathrm{Mag}(Z) = \lambda \mathrm{Mag}(A) + (1-\lambda)\mathrm{Mag}(R) \quad (1)$$

where Mag(A) and Mag(R) are the magnitude of a fuzzy number $N=(n_1,n_2,n_3)$ defined by

$$Mag(N) = \frac{1}{12}\left(n_1 + 10n_2 + n_3\right) \quad (2)$$

3. Proposed AHP Method

The following are the proposed steps for developing the AHP model:
Step 1: Construct pairwise comparison matrices (PCM) for the restriction and reliability.
Step 2: Aggregate the PCM separately using the geometric mean operator.
Step 3: Sum up the aggregated triangular fuzzy numbers of both components for Z-numbers and calculate their fuzzy weights.
Step 4: Calculate the magnitude of Z-numbers (Farzam et al., 2021) using the formula (1).
Step 5: Normalize the obtained magnitude to obtain the final criteria weight.

4. Prioritization of Public Services

For the validation of the proposed AHP method, the criteria ranking for prioritization of public services discussed in Sergi and Sari (2021) is adopted. Six criteria are considered: reduced cost (C_1), fast response (C_2), ease of accessibility (C_3), reduced service time (C_4), improved information availability (C_5) and increased quality (C_6). Using several values for the parameter λ, the obtained ranking is $C_3 \succ C_4 \succ C_2 \succ C_1 \succ C_6 \succ C_5$.

Table 1
Final criteria weights

Criteria	λ					
	0.5	0.6	0.7	0.8	0.9	1.0
C_1	0.1324	0.1242	0.1161	0.1082	0.1003	0.0925
C_2	0.1716	0.1721	0.1727	0.1733	0.1739	0.1744
C_3	0.2961	0.3153	0.3343	0.3531	0.3716	0.3900
C_4	0.2223	0.2314	0.2404	0.2493	0.2581	0.2668
C_5	0.0710	0.0615	0.0521	0.0429	0.0338	0.0247
C_6	0.1067	0.0954	0.0843	0.0732	0.0624	0.0516

5. Conclusion

The implementation of Z-numbers in any MCDM methods must consider the nature of restriction and reliability components to preserve the decision information. The concept magnitude of Z-numbers was integrated with the AHP to produce a consistent criteria ranking. In the proposed model, the restriction and reliability components of Z-numbers were combined using the magnitude formula to determine the priority weights. This method does not only preserve the initial information in form of Z-numbers, but also simplifies the calculation involving Z-numbers. However, this research is limited to criteria ranking using the proposed AHP model. Hence, there is a need to integrate the AHP model with other MCDM methods such as TOPSIS or VIKOR to help decision makers in ranking the alternatives.

6. Key References

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7. Appendices

Table 2
Pairwise comparisons for the restriction of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	EI	RWI	RMI	RMI	GI	WI
C_2	WI	EI	RWI	RWI	GI	MI
C_3	MI	WI	EI	WI	AI	MI
C_4	MI	WI	RWI	EI	GI	MI
C_5	RGI	RGI	RAI	RGI	EI	RWI
C_6	RWI	RMI	RMI	RMI	WI	EI

Table 3
Pairwise comparisons for the reliability of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	AR	VWR	FR	FR	VHR	VHR
C_2	VHR	AR	VWR	VWR	VHR	FR
C_3	FR	VHR	AR	VHR	SR	FR
C_4	FR	VHR	VWR	AR	VHR	FR
C_5	VWR	VWR	SU	VWR	AR	VWR
C_6	VWR	FR	FR	FR	VHR	AR

Table 4
Linguistic values for the restriction matrix (Sergi & Sari, 2021)

Linguistic Terms	Triangular Fuzzy Number
Equally important (EI)	(1,1,1)
Weakly important (WI)	(1,3,5)
Moderately important (MI)	(3,5,7)
Greatly important (GI)	(5,7,9)
Absolutely important (AI)	(7,9,9)
Reciprocal weakly important (RWI)	(1/5,1/3,1)
Reciprocal moderately important (RMI)	(1/7,1/5,1/3)
Reciprocal greatly important (RGI)	(1/9,1/7,1/5)
Reciprocal absolutely important (RAI)	(1/9,1/9,1/7)

Table 5
Linguistic values for the reliability matrix (Sergi & Sari, 2021)

Linguistic Terms	Triangular Fuzzy Number
Absolutely reliable (AR)	(1.0,1.0,1.0)
Strongly reliable (SR)	(0.7,0.8,0.9)
Very highly reliable (VHR)	(0.6,0.7,0.8)
Highly reliable (HR)	(0.5,0.6,0.7)
Fairly reliable (FR)	(0.4,0.5,0.6)
Weakly reliable (WR)	(0.3,0.4,0.5)
Very weakly reliable (VWR)	(0.2,0.3,0.4)
Strongly unreliable (SU)	(0.1,0.2,0.3)
Absolutely unreliable (AU)	(0.0,0.1,0.2)

Table 6
Aggregated pairwise comparison matrix

Criteria	Restriction Part	Reliability Part
C_1	(0.523,0.809,1.308)	(0.508,0.605,0.698)
C_2	(0.918,1.506,2.608)	(0.485,0.583,0.677)
C_3	(1.995,3.557,4.718)	(0.586,0.679,0.769)
C_4	(1.442,2.365,3.608)	(0.508,0.605,0.698)
C_5	(0.177,0.218,0.323)	(0.305,0.415,0.515)
C_6	(0.289,0.447,0.755)	(0.475,0.572,0.665)

Table 7
Summation of aggregated pairwise comparison matrix and its inverse

	Restriction Part	Reliability Part
Summation	(5.344,8.902,13.32)	(2.868,3.459,4.021)
Inverse	(0.075,0.112,0.187)	(0.249,0.289,0.349)

Table 8
Fuzzy weights for all criteria

Criteria	Restriction Part	Reliability Part
C_1	(0.039,0.091,0.245)	(0.126,0.175,0.243)
C_2	(0.069,0.169,0.488)	(0.121,0.169,0.236)
C_3	(0.150,0.400,0.883)	(0.146,0.196,0.268)
C_4	(0.108,0.266,0.675)	(0.126,0.175,0.243)
C_5	(0.013,0.025,0.061)	(0.076,0.120,0.180)
C_6	(0.022,0.050,0.141)	(0.118,0.165,0.232)

Table 9
Magnitude of triangular fuzzy numbers

Criteria	Restriction Part	Reliability Part
C_1	0.0994	0.1766
C_2	0.1874	0.1701
C_3	0.4190	0.1981
C_4	0.2867	0.1766
C_5	0.0266	0.1213
C_6	0.0554	0.1670

Table 10
Unnormalized criteria weights

Criteria	λ					
	0.5	0.6	0.7	0.8	0.9	1.0
C_1	0.1380	0.1302	0.1225	0.1148	0.1071	0.0994
C_2	0.1788	0.1805	0.1822	0.1839	0.1857	0.1874
C_3	0.3085	0.3306	0.3527	0.3748	0.3969	0.4190
C_4	0.2316	0.2426	0.2536	0.2646	0.2757	0.2867
C_5	0.0740	0.0645	0.0550	0.0455	0.0361	0.0266
C_6	0.1112	0.1001	0.0889	0.0777	0.0666	0.0554