An econometric model MLRM + Analytic Hierarchy Process (AHP) to select the best alternative for a real estate investment. Case: Apartment in Panama City, Panama.

Miguel Camacaro Pérez[1], Ernesto Mock Díaz[2]

[1] *Appraisers Reading Club*, Venezuela -mundovalor@gmail.com.
[2] *Appraisers Reading Club*, Panamá - proavaluos@yahoo.com

ABSTRACT

A person interested in making a real estate investment, starts a search process through real estate websites, finding several alternatives: real estate products differentiated in a range of prices that fit his purchasing power. In order to assist the future buyer in the decision making process, the real estate consultant must evaluate the most important characteristics of each alternative in order to offer a balanced quality-price ratio, in the best case, using a selection procedure with empirical basis. This article presents a case study for the selection of an apartment located in a sector of Panama City (Panama), for which offers with a price range between [165,000 and 215,000]USD were analyzed, using one of the multiple attribute decision making method (MADM), the Analytic Hierarchy Process (AHP). The criteria or variables considered in the multi-criteria method were: area, parking space, age, quality, building floor and distance to the value pole, which, according to the econometric model of a sample of 97 offers, were statistically significant in explaining the total real estate prices. The entire procedure established for applying the AHP was followed, but the Saaty Table was not used to structure the pairwise comparison matrix of the criteria. For the determination of the eigenvector of the criteria, the importance of the intensity in the fundamental scale was replaced by the ratio between the standardized coefficients (t^*) of each independent variable of the econometric model. The ranking of the overall weighting vector suggested that the best option was PH Coral Tower, which coincided with the solution provided using the Saaty table, but with more robust results, thus mitigating the subjectivity of the heuristic solution provided by the original AHP procedure.

Key words: model, econometrics, multi-criteria, real estate, appraisal.

1. INTRODUCTION

In the real estate business, there is always an investment decision to be made. The industrialist must decide where to install his industry, the real estate developer where to locate his next real estate development or the hotel entrepreneur must choose where to build his recreational facilities. In order to decide if the investment is viable, one of the main considerations is the economic-financial evaluation of the feasibility of the undertaking, which is fundamental.
At a lower investment level, one of the biggest decisions a person, or a family group, must make is when they must choose between several alternatives for the acquisition of a property. For many, it may be the only major investment in their lifetime. The real estate market is a market of imperfect competition, characterized by the heterogeneity of the real estate product, which makes it difficult to estimate prices and, therefore, to select the ideal property in a negotiation.
Real estate brokers, among others, are involved in real estate brokerage, who manage a real estate portfolio and offer the corresponding advice for the analysis of the best alternative that fits the buyer's purchasing power and satisfies the buyer's needs. In order to choose among the alternatives, in the best of cases, the agents and the client use multiple criteria to base the decision making process, and this is also contemplated for large real estate businesses when analyzing the viability of the undertaking..
The scientific discipline called Operations Research has a branch that explicitly evaluates multiple criteria, which is the Multiple Criteria Decision Making (MCDM), being one of the consecrated methods, the Analytic Hierarchy Process (AHP) developed by Thomas Saaty in the 70's and 80's. Since its appearance, the method proposed by Saaty and his collaborators, has been highly praised and popular among its users, but the scientific community has also criticized it. Since its appearance, the method proposed by Saaty and his collaborators has been highly praised and popular among its users, but it has also been criticized by the scientific community, which has resulted in the generation of an extensive bibliography.[1].
Regarding the opinion against AHP held by some researchers, Harker & Vargas (1987) assert that the main criticisms that have been made of AHP are not valid, because AHP is based on a solid theoretical foundation and, as the examples in the literature and the daily operations of several government agencies, companies and consulting firms show, it is a viable and usable tool for decision making. However, this does not mean that AHP is the method that can solve all decision-making problems.
Author Kujawski (2003) has pointed out that much of the popularity of AHP is due to: (1) the appearance of a scientific approach due to the use of a matrix formulation; (2) the attractive technique of eliciting weights and scores based on pairwise comparison using a verbal scale; and (3) the existence of a readily available software tool.
Saaty&Vargas (2011) admit that one of the issues discussed referring to AHP, is the use of different scales to translate judgments into ratios, they consider that pairwise reciprocal comparisons are used to express judgments in a semantic way, automatically linking them to a fundamental numerical scale of absolute numbers from which the principal eigenvector of priorities is derived..
Regarding the use of AHP and the fundamental value scales of the Saaty Table, Velasquez&Hester (2013), indicate that the method is easy to use; it is scalable; the hierarchical structure can be easily adjusted to fit problems of many sizes; it does not require much data, but

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1 *Emrouznejad&Marra* (2017) present a paper evidencing the growing body of work on AHP published between 1979 and 2017. Given the large number of papers in this field (8,441 published pieces), we opted for a quantitative analysis, based on scientometric mapping and SNA.

has problems due to the interdependence between criteria and alternatives; it can lead to inconsistencies between judgment and ranking criteria; even inversion of the ranking.

For Kadziński & Tervonen (2013), when analyzing the decision maker (Decision Maker - DM) it is observed that it provides indirect preference information in the form of pairwise comparisons of reference alternatives, which derive in a set of compatible value functions. Now combining these results with the results from the use of robust ordinal regression, extreme range analysis and stochastic multicriteria acceptability analysis, it is evident that they complement each other in a unified decision support framework.

Franek&Kresta (2014), investigated the application and characteristics of different rating scales developed by academics for use in AHP, according to the results presented, the linear scale (Saaty) remains a favorable option. Depending on greater consistency, priority values and selection of the most important criterion a DM may use the quadratic, logarithmic, power or geometric scales to clearly highlight the preferred criterion. Future research should focus on the use of different scales in different decision-making problems.

In a study by Meesariganda & Ishizaka (2017) indicated that the Saaty scale was not the best scale for any participant in the evaluation conducted. Using cloud computing they tested a new method of map scaling, generating better scales with the algorithm proposed in this research work.

Asadabadi, M. et al (2018) in their analytical study on one of the MCDM methods, called Analytical Hierarchical Process (AHP) demonstrate that AHP is very likely to provide a ranking of options that would not be acceptable to a rational person.

Gopel (2019) performs a new approach to compare different scaling functions and derive a recommendation for the application of scales. The approach is based on simple analytical functions and takes into account the number of criteria of the decision problem. A generalization of the so-called balanced scale is proposed and a new adaptive balanced scale is introduced.

In a study by Khan, A. U., & Ali, Y. (2020), it was concluded that AHP has dominated the last 20 years in terms of number of publications in all major categories, i.e., engineering/technology/applied sciences, social sciences, health studies, and environmental studies. This shows that AHP produces more authentic and reliable results and has been preferred by researchers.

Dos Santos et al (2021), conducted a bibliometric study of publications on the subject of multiple criteria and decision, proving that the AHP method is the most used in all areas of knowledge. They analyzed three warship projects in relation to nine operational and economic criteria, and concluded that the best alternative would be the construction of a new ship, based on AHP, and then this option was endorsed through sensitivity analysis that allows obtaining the weights and ordering the alternatives through the Gaussian factor, without the need to apply pairwise comparisons of alternatives and criteria.

The bibliographic review made for this article was not exhaustive; however, it is representative of the positions of the scientific community on the use of the fundamental scale proposed by Saaty. This article presents an alternative for the replacement of the scale of the values of Saaty's table by the ratio between the standardized coefficients (t*) of each independent variable of an econometric model.

Thus, the Methodology section presents the theoretical bases of the Direct Comparative Method of Market Data, based on the Brazilian appraisal standards ABNT NBR 14653-2, the Analytic Hierarchy Process (AHP) and the development of the proposal for the use of the results of the Multiple Linear Regression Model (MLRM) in the AHP. In the Results section, the calculations performed for each procedure analyzed are presented. Then, the Discussion and Conclusion sections are presented, where the formality of the methodological proposal resulting from the research work is presented.

2. METHODOLOGY

For the development of this article it is very important that the theoretical basis on the scientific treatment of the Comparative Market Data Method, the procedure of the Analytic Hierarchy Process (AHP) method and the substitution of empirically based comparison elements estimated using the Multiple Linear Regression Model (MLRM) by the heuristic comparison elements of the AHP method are presented.

2.1 Direct Comparative Market Data Method

Item 8.2.1.4.3 of the ABNT[2] NBR 14653-2, refers to scientific treatment and states that whatever models are used to infer market behavior and value formation, their assumptions must be properly explained and tested.

When talking about factors that affect the market value of real estate, it is necessary to use a model, and this is nothing more than a simplified representation of the market reality. Econometric models are a particular class of models that aim to quantify relationships between variables based on economic laws that support them.

In a broad sense, it can be said that econometric analysis is concerned with developing models through which hypotheses concerning economic systems can be verified.

The models that relate the price of a property to its characteristics are known as hedonic price models, which can be expressed by the following function:

$$P = f(L, E, T) + \varepsilon \qquad (1)$$

Where:

- P : price of the good as a function of its attributes.
- f : functional form.
- L : location variables.
- E : structural variables.
- T : time variables.
- ε: errors.

In order to explain the behavior of real estate prices based on a population of m observed prices (Y_i) considering k influential characteristics ($X_{ij},\ j=1,\dots,k$) the multiple linear regression model is used:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \dots + \beta_k X_{ik} + \varepsilon_i;\ i=1,\dots,m \qquad (2)$$

Where:

- $Y_1,\dots,Y_m$: dependent variables (price of real estate)
- $X_{1k},\dots,X_{mk}$: independent variables (intrinsic and extrinsic characteristics of the real estate)..
- $\beta_0,\dots,\beta_k$: model parameters (attributes or shadow prices).
- $\varepsilon_1,\dots,\varepsilon_m$: random errors of the model.

It is generally not economically or temporarily feasible to obtain all the market data from a population, so we work with a subset of n elements of this population, called a sample. In the linear regression model for market representation, the dependent variable is expressed as a linear

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[2] ABNT acronym of the Brazilian Association of Technical Standards (Associação Brasileira de Normas Técnicas), the body responsible for standardization in Brazil. The acronym NBR refers to Brazilian Standards, in this case, NBR-14653 Part 2. Urban Property Appraisals.

combination of the independent variables, in original or transformed scale, and the respective estimates of the population parameters, including errors, which originate from:

- Effects of undetected variables and important variables not included in the model
- Accidental imperfections of observation or measurement.
- Variations in human behavior, such as: different negotiation skills, desires, needs, compulsions, whims, anxieties, differences in purchasing power, cultural differences, among others.

Based on the sample taken from the market, the estimation of the parameters of the regression model is done using the least squares or maximum likelihood method. The modeling should state the hypotheses regarding the behavior of the dependent and independent variables, based on the appraisal engineer's knowledge of the market, when the null or alternative hypotheses are formulated for each population parameter.

When regression models are used, the content of the standard emphasizes the need to observe basic assumptions, mainly regarding normality, homoscedasticity, non-multicollinearity, non-autocorrelation, independence or non-existence of outliers and specification errors, in order to obtain unbiased, efficient and consistent appraisals.

2.2 Analytic Hierarchy Process –AHP

Individuals or groups that must make decisions when seeking to solve a situation or problem analyze quantitative and qualitative parameters using multiple criteria to select the best possible solution among the existing alternatives. Decision-making processes involve a series of steps: identifying problems, constructing preferences, evaluating alternatives, and determining the best alternatives (Simon 1977; Keendy and Raiffa 1993; Kleindorfer, Kunreuther, and Schoemaker 1993 in Tzeng and Huang 2011).

In general, three types of formal analysis can be used to solve decision-making problems: descriptive, prescriptive and normative. The latter two are dealt with in the fields of Decision Science, Economics and Operations Research (OR), the latter being an applied science aimed at solving real problems, using methods from other scientific areas to introduce elements of objectivity and rationality in the decision-making process.

There is a branch of Operations Research that explicitly evaluates multiple criteria, which is Multiple Criteria Decision Making (MCDM). Whether the decision maker is an individual, a group or managers, MCDM helps to make decisions about a complex problem by evaluating and choosing alternatives to solve it using different criteria and points of view (Kadziński & Tervonen, 2013).

In general, the procedure follows six steps, which include (1) problem formulation, (2) identify requirements, (3) set objectives, (4) identify various alternatives, (5) develop criteria and (6) identify and apply the decision making technique (Sabaei, Erkoyuncu, & Roy, 2015).

In Yepes (2018) to facilitate systematic research in the field of MCDM, Hwang and Yoon (1981) suggested that MCDM problems can be classified into two main categories: multiple attribute decision making (MADM) and multiple objective decision making (MODM), based on the different purposes and types of data.

It is in the interest of the authors of this article to refer to MADM methods. The classification of these methods is made depending on the type of initial information (deterministic, stochastic or uncertain), or depending on the groups of decision-makers (a single group or several groups). The most common classification is the one proposed by Hajkwociz and Collins (2007) and De Brito and Evers (2016), which can be seen in the following table:

Table 1: Classification of MADM methods

Group MADM	MADM'Method
Direct scoring methods	*Simple Additive Weighting* (SAW)
	Complex Proportional Assessment (COPRAS)
Distance-based methods	*Goal Programming* (GP)
	Compromise Programming (CP)
	Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).
	Multicriteria Optimization and Compromise Solution (VIKOR).
Pairwise comparison methods	*Analytic Hierarchy Process* (AHP)
	Analytic Network Process (ANP)
	Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH).
Overcoming methods	*Preference Ranking Organization Method for Enrichment of Evaluations* (PROMETHEE).
	Elimination and Choice Expressing Reality (ELECTRE)
Methods based on utility or value functions	*Multi-attribute Utility Theory* (MAUT)
	Multi-Attribute Value Theory (MAVT)
	Integrated Value Model for Sustainable Appraisals (IVMSA).

Penadés-Plà et al., (*2016*): information on the classification of Table 1

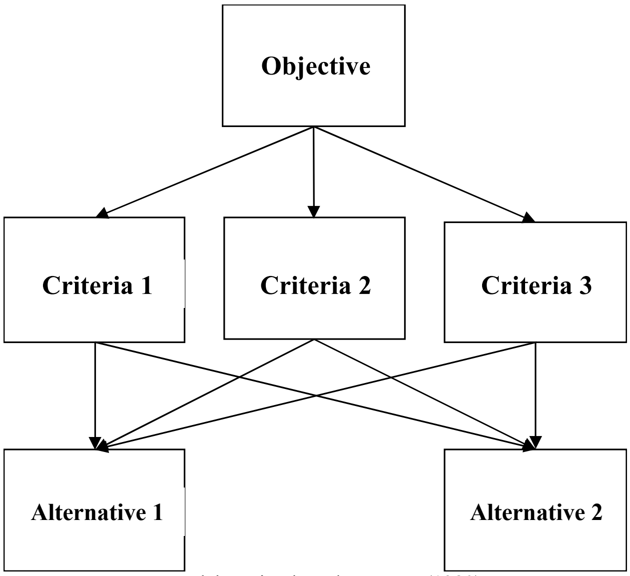
Regarding pairwise comparison methods, Thomas Saaty introduced the Analytic Hierarchy Process (AHP) in the 1970s and the Analytic Network Process (ANP) more recently. Among his co-authors and colleagues are Ernest Forman and Luis Vargas. Saaty is one of the most successful people in the field of information management and communications in disseminating his method for complex decision making.

According to Moreno Jiménez *et al* (2019), the Analytic Hierarchical Process (AHP) is a Multicriteria decision technique that combines tangible and intangible aspects to obtain, on a ratio scale, the priorities associated with the alternatives of the problem. In an environment of certainty, the AHP provides the possibility of including quantitative data related to the decision alternatives. The advantage of the AHP is that it additionally allows the incorporation of qualitative aspects that are usually left out of the analysis due to their complexity to be measured, but that may be relevant in some cases.

The main features of this approach are modeling the problem using a hierarchical structure, using paired comparisons to incorporate the decision maker's preferences, and obtaining a ratio scale that is valid for complex decision making. According to Saaty, making a decision in an organized way to generate priorities requires decomposing the decision into the following steps:

1. Define the problem and determine the type of knowledge sought.
2. Structure the decision hierarchy from the top with the decision objective, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which is usually a set of alternatives), as can be seen in the figure below:

Figure 1: Hierarchical structure of AHP



Own elaboration based on Saaty (1980).

3. Construct a set of pairwise comparison matrices. Each element of a higher level is used to compare the elements of the level immediately below with respect to it.
4. Use the priorities obtained from the comparisons to weight the priorities at the next lower level. This is done for each item. Then, for each item at the lower level, sum their weighted values and obtain their overall or global priority. Continue this process of weighting and summing until you get the final priorities of the alternatives at the lowest level.

For the mathematical development of the AHP, the work presented by Tzeng & Huang (1981) will be cited. If we wish to compare a set of d n attributes in pairs according to their relative importance weights, where attributes are denoted as $a_1, a_2, \ldots, a_n$ and the weights are denoted $w_1, w_2, \ldots, w_n$, pairwise comparisons can be represented by means of subjective perception questionnaires such as:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix}, \quad (3)$$

Where $a_{ij} = 1/a_{ji}$ (positive reciprocal) and $a_{ij} = a_{ik} / a_{jk}$. Please note that, in realistic situations, w_i/w_j is usually unknown. Therefore, the problem for the AHP is to find a_{ij} such that $a_{ij} \cong w_i/w_j$, let be a matrix of weights represented as:

$$W = \begin{matrix} & w_1 & \cdots & w_j & \cdots & w_n \\ w_1 & w_1/w_1 & \cdots & w_1/w_j & \cdots & w_1/w_n \\ \vdots & \vdots & & \vdots & & \vdots \\ w_i & w_i/w_1 & \cdots & w_i/w_j & \cdots & w_i/w_n \\ \vdots & \vdots & & \vdots & & \vdots \\ w_n & w_n/w_1 & \cdots & w_n/w_j & \cdots & w_n/w_n \end{matrix}, \quad (4)$$

Multiplying W by w gives:

$$W.w = \begin{matrix} \\ w_1 \\ \vdots \\ w_i \\ \vdots \\ w_n \end{matrix} \begin{matrix} w_1 & \cdots & w_j & \cdots & w_n \\ \left[\begin{matrix} w_1/w_1 \\ \vdots \\ w_i/w_1 \\ \vdots \\ w_n/w_1 \end{matrix}\right. & \begin{matrix} \cdots \\ \\ \cdots \\ \\ \cdots \end{matrix} & \begin{matrix} w_1/w_j \\ \vdots \\ w_i/w_j \\ \vdots \\ w_n/w_j \end{matrix} & \begin{matrix} \cdots \\ \\ \cdots \\ \\ \cdots \end{matrix} & \left.\begin{matrix} w_1/w_n \\ \vdots \\ w_i/w_n \\ \vdots \\ w_n/w_n \end{matrix}\right] \end{matrix} \begin{bmatrix} w_1 \\ \vdots \\ w_j \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ \vdots \\ w_j \\ \vdots \\ w_n \end{bmatrix} \quad (5)$$

Table 2: Saaty's Fundamental Scale

Intensity	1	3	5	7	9	2, 4, 6, 8
Linguistic	Equal	Moderate	Strong	Demonstrated	Extreme	Intermediate values

Own elaboration based on Tzeng & Huang (1981).

Or

$$(W - nI)\, w = 0 \quad (6)$$

Table 2 represents the fundamental scale used to compare the weight of importance between the criteria according to the linguistic meaning from 1 to 9 to denote from equal importance to extreme importance.

Since solving equation 6 is an eigenvalue problem, we can derive the comparative weights by finding the eigenvector w with its respective $\lambda_{\max}$ that satisfies $Aw = \lambda_{\max}\ w$, where $\lambda_{\max}$ is the largest eigenvalue of the matrix A, i.e. find the eigenvector w whit λ_{max} respective for $(A\text{-}\lambda_{max} I)w = 0$. In addition, to ensure the consistency of subjective perception and the accuracy of comparative weights, two measures are suggested, the consistency index (***C.I.***) y the consistency ratio (***C.R.***). The ***C.I.*** equation can be expressed as:

$$C.I. = \frac{(\lambda_{max}) - 1}{(n) - 1} \quad (7)$$

Where $\lambda_{\max}$ is the largest eigenvalue and n denotes the number of attributes. On the other hand, the ***C.R.*** can be calculated as:

$$C.R. = \frac{C.I.}{R.I.} \quad (8)$$

Where ***R.I.*** is a random index that indicates the consistency of a given random matrix according to the following table:

Table 3: Random index (RI) values for a discrete set of criteria n ≤ 10

Criteria numbers	2	3	4	5	6	7	8	9	10
RI	0	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49

Own elaboration based on Sánchez-Garrido (2022).

According to Sánchez-Garrido (2022), if *C.I.* is close to ***R.I.***, the matrix has been completed randomly, thus expressing an absolute inconsistency in the evaluation of the problem to be solved. Conversely, a low consistency ratio *C.R.* means that the DM has a clear knowledge of the problem to be solved, being for *C.I.* = 0 a complete consistency. Inconsistency will be acceptable if the *C.R.* does not exceed the values indicated in Table 4:

Table 4: Maximum Consistency Ratio (C.R.)

Matrix size **[*n*]**	***C.R.*** **[%]**
2	5
4	9
≥5	10

Own elaboration based on Sánchez-Garrido (2022).

If a matrix exceeds the maximum *C.R.*, according to Saaty, the weights must be revised to improve consistency and this can be done in two ways: the first consists of classifying the activities in a simple order based on the weights obtained with the matrix proposed, and developing, taking into account the knowledge of the previous categorization, a second matrix of pairwise comparison. The second way is through the application of goal programming.

2.3 From the Multiple Linear Regression Model (MLRM) and the AHP

When analyzing the explanatory variables that are part of an MLRM, it is interesting to know which variable has the greatest influence on the variation of the response variable, and this is possible by comparing the standardized coefficients (t^*) of the model. The variable with the highest coefficient value (in absolute value) is the one that has the greatest influence on the response variable; therefore, the relevance relationship between the variables can be determined.

Let the following sample regression equation be:

$$\widehat{Y_i} = b_0 + b_1 X_{i1} + b_2 X_{i2} + b_3 X_{i3} + \ldots + b_k X_{ik} \; ; \; i = 1,\ldots,m \tag{9}$$

Where:

- $\widehat{Y_i}$: dependent variable (estimated price of the real estate)
- $X_{1k},\ldots,X_{mk}$: independent variables (intrinsic and extrinsic characteristics of the real estate).
- $b_0,\ldots,b_k$: are the estimators of the model parameters.

The standardized coefficient is defined as (t_k^*) as the quotient of the value of the model parameter estimator b_k between its standard deviation $s(b_k)$:

$$t_k^* = \frac{b_k}{s(b_k)} \tag{10}$$

Now, the standard deviation of the coefficient $s(b_k)$ has the following equation:

$$s(b_k) = S_e \frac{1}{\sqrt{(X_i - \bar{X})^2}} \tag{11}$$

Where:
X_i : is the independent variable
$\bar{X}$: average of the independent variable
Se : the standard deviation of the model

The equation of the standard deviation of the *Se* model is as follows:

$$s_e = \sqrt{\frac{\Sigma\left(Y_i - \hat{Y}_i\right)^2}{n - k - 1}} \quad (12)$$

Where:
X_i : is the independent variable
$\bar{X}$: average of the independent variable
n : sample size
k : number of independent variables

In order to make a decision on the selection of a real estate investment using the multi-criteria AHP method, once a sample of the real estate market has been analyzed and the Multiple Linear Regression Model (MLRM) has been obtained, the importance of the intensity in the absolute scale of the Saaty Table in the pairwise comparison matrix of the criteria is replaced by the relevance ratio between the standardized coefficients (t_k*) of each independent variable of the econometric model.

Thus, the comparison matrix is structured considering the absolute values and the standardized coefficients:

$$A = \begin{bmatrix} t_1^*/t_1^* & \cdots & t_1^*/t_3^* & \dots & t_1^*/t_k^* \\ \vdots & & \vdots & & \vdots \\ t_3^*/t_1^* & \cdots & t_3^*/t_3^* & \dots & t_3^*/t_k^* \\ \vdots & & \vdots & & \vdots \\ t_k^*/t_1^* & \dots & t_k^*/t_3^* & \dots & t_k^*/t_k^* \end{bmatrix} \quad (13)$$

Once the matrix of comparisons ***A*** of the variables has been obtained, we proceed to calculate the eigenvector of the matrix, the λ_{max}, the consistency index (***C.I.***) and the consistency ratio (***C.R.***). There being no inconsistency problems, we proceed to calculate the eigenvectors of the alternatives by normalization by the sum of the values of the (quantitative) variables.
Saaty & Vargas (2011) point out that, if the alternatives are measured on a different scale for each criterion, it is evident that standardization is the instrument that provides the structural effect to update the importance of the criteria according to the alternatives available.
Once the vectors of the variables and alternatives have been determined, the vector of general weightings is calculated, from which the ranking will be extracted to select the alternative that is placed in first place[3].

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[3] It is advisable, as part of this analysis, to calculate the eigenvector of the criteria using the values of the fundamental scale of the Saaty table, in order to have elements for comparison with the results obtained by the standardized coefficients of the MLRM.

3. RESULTS

It is very important to separate the results of each of the methods or procedures for their individual analysis, and then present the combination of results by substituting the items of the Saaty Table scale for the MLRM test statistics.

3.1 Direct Comparative Market Data Method

The population under study is represented by multi-family residential units, with areas ranging from [60,00 y 140,00] m^2, located in buildings that have between 10 y 40 floors, located in the Corregimiento de Rio Abajo area of Panama City.
For the structuring of the Multiple Linear Regression Model (MLRM), a sample with size *n* equal to 97 data corresponding to prices of market offers of multi-family residential units was raised. The physical boundaries of the polygonal analyzed are Vía Simón Bolívar or Transístmica, Río Abajo, Corredor Sur Highway (Pacific Ocean) and Vía Brasil.
For the application of statistical inference (multiple linear regression analysis), the selection of variables was made based on the characteristics of the taxable property and its urban environment, which are classified in the following table:

Table 5: Variables of the econometric model

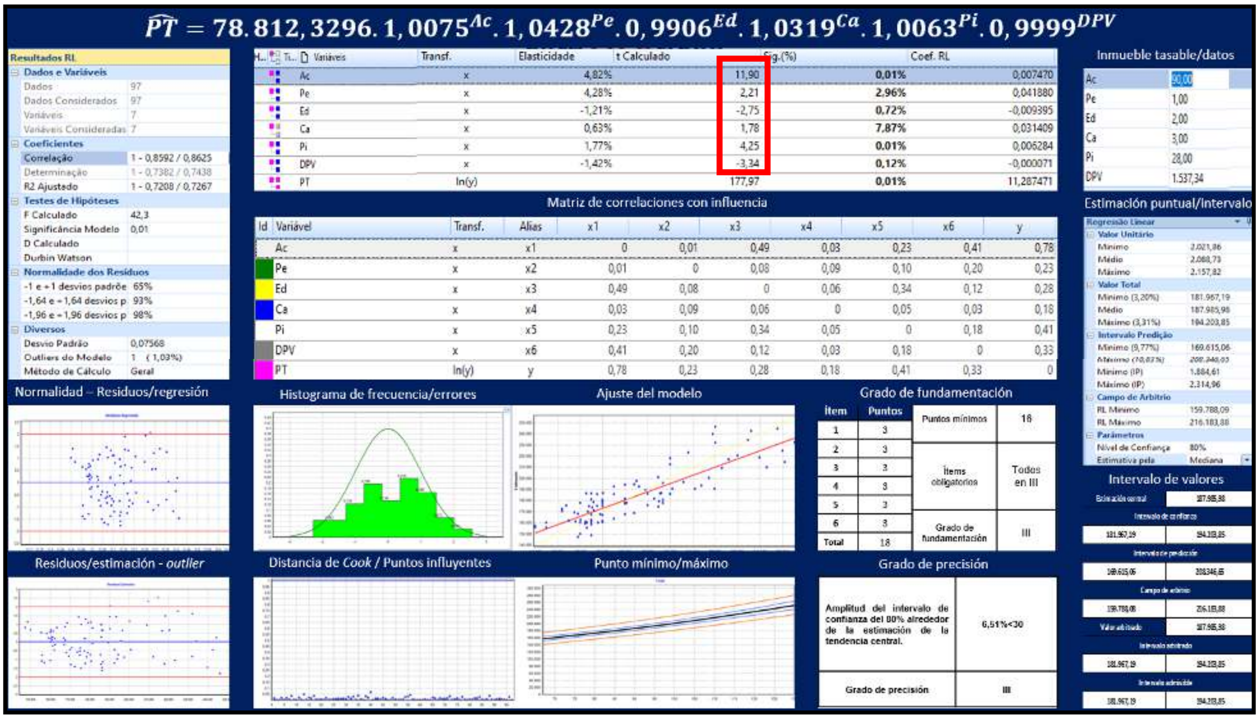
Variable	IM	Unit	Description		Variable Type	Measurement Scale
Construction area	*Ac*	m^2	Legal or assessable area of the property.		Explanatory Quantitative Continuous	Reason
Parking place	*Pe*	adim	Number of parking spaces in the property.		Explanatory Quantitative Discrete	Reason
Age	*Ed*	year	Age of the building where the property is located.		Explanatory Quantitative Discrete	Reason
Quality[4]	*C*	adim	High	3	Explanatory Categorical Discrete	Ordinal
			Medium High	2		
			Medium	1		
Floors	*Pi*	adim	Number of floors in the building		Explanatory Quantitative Discrete	Reason
Distance to the valorizing pole	*DPV*	m	Distance from the building to the central point of the Omar Recreational and Cultural Park.		Explanatory Proxy Continuous	Reason
Total price of the property	*PT*	UM	Total price of the apartment offers.		Explained Quantitative Continuous	Reason

Own elaboration based on Camacaro & Mock (2021)

[4] Precisely AHP is an important alternative for the qualification of buildings to optimize the categorical variable used in this model. The procedure can be observed in the work of Celso José Gonçalves (2021) presented at COBREAP-Brazil. https://ibape-nacional.com.br/biblioteca/author/admin/page/7/

In the following figure called «***MLRM Chart***»[5], is presented in a mosaic of econometric modeling results:

Figure 2: MLRM Chart (mosaic of results)



Own elaboration based on Camacaro & Mock (2021)

From figure 2, it can be commented that an estimation equation of the multiplicative type was obtained. The standard states that in order to reach **Degree of substantiation** III, 6 (k+1) must be less than the number of data used; here it is 6*(6+1) = 42 less than 97, so this condition is met. There were also no problems of **micronumerosity** in the qualitative variables.

The **correlation coefficient (*r*)** is equal to 0.8592, indicating a strong correlation between the explained variable and the explanatory variables. The **coefficient of determination (R^2)** is equal to 0.7382, indicating that 73.82% of the variation of the unit values around the mean value is explained by the independent variables included in the model.

As for the interpretation of the significance of the regressors of the explanatory variables, all of them have a significance of less than 10%, which allows us to classify this item in **Degree of substantiation** III. The **standardized coefficients** for each variable that will be used to replace the fundamental scale of Saaty's table are highlighted in a red frame.

The **calculated** F is equal to 42.30; while the significance level of the model is 0.01%, which is less than the α significance level, so the null hypothesis is rejected and at least one of the independent variables included in the model is important for the explanation of the variability of the prices observed in the market, the **Degree of substantiation** III is reached.

Regarding the **Normality** of the errors, comparing the relative frequency of the standardized errors of the sample in the intervals of [-1;1], 65% [-1.64;1.64] 93% and [-1.96;1.96], 98% with probabilities of the standard normal distribution in the same intervals, i.e. 68%, 90% and 95%, there is an approximation considering the tolerance for this test.

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[5] They are images of the output of the SisDEA program https://pellisistemas.com/software/sisdea-avaliacao-de-imoveis/

The **standard deviation** of the regression model is equal to 0.07568. The presence of an **outlier** (1.03%) is proportionally lower than the 5% tolerable for this analysis of **normality** in the residuals. When reviewing the values of the correlations, it is evident that there are no high correlations between the variables (less than 0.80), which explains that there is no presence of **multicollinearity** between the independent variables.
Continuing with the graphical tests on **Normality** and **Homoscedasticity** in the errors, the scatter plot of the values estimated by the model versus the standardized errors (of the estimation and regression) shows the distribution of the errors between -2 and 2, which corroborates the approximate normality of the errors (the outlier is also observed).
While the cloud of points does not show any trend, which is an indication that they are randomly arranged and it is interpreted that the model errors have a **homocedastic** behavior, i.e., the errors have constant variance.
The histogram of the model errors complements the information on the approximate **normality** in graphical form by comparing it with the normal distribution.
The presence of influential points in the model can be checked using the **Cook's Distance** plot, in this case it was observed that there are no influential points among the sample data of the regression model.
The fit of the model ́ must be verified using the dispersion graph of the prices observed on the abscissa axis and the values estimated by the model on the ordinate axis, which must contain points close to the bisector of the first quadrant, as shown can be seen in this graph the adjustment made by the regression model is verified.
When observing the graphs there is no evidence of **maximum and minimum points**, therefore, the model has no restrictions for its use. It is concluded that the model complies with the specifications of the NBR 14653-2 Standard and its report, based on the estimation results, reaches a **Degree of substantiation** III and **Degree of Precision** III.
In the **MLRM Chart**, information related to the point estimate and by interval with the data of a property is included, highlighting the confidence interval, the prediction interval and the decision interval, all contained in the required Allowable Value Intervals table in Annex A of the NBR 14653-2 Standard.

3.2 Analytic Hierarchy Process -AHP

From the sample of properties offered for statistical analysis through MLRM, a random choice of four properties was made to select the best alternative according to the ranking of AHP results:

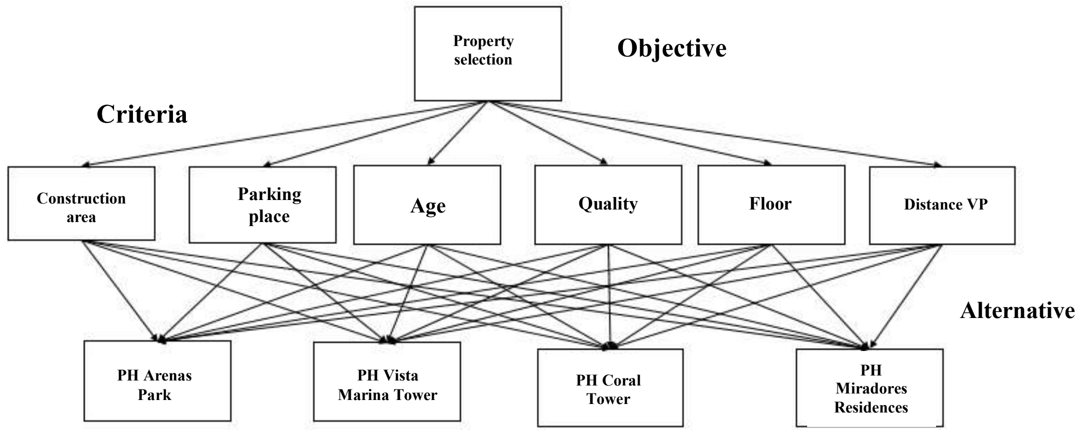
Table 6: Random sample of offers with total prices

Property	Construction area [m^2]	Parking Place [adim]	Age [year]	Quality [adim]	Floor [adim]	Distance to the valorizing pole [m]	Total price [UM]
PH Arenas Park	67	1	5	2	23	1.157,96	165.000,00
PH Vista Marina Tower	105	2	8	1	16	1.541,94	170.000,00
PH Coral Tower	81	2	5	3	26	782,71	200.000,00
PH Miradores Residences.	90	2	2	2	28	1.537,34	168.500,00

Own elaboration based on Camacaro Mock (2021)

Based on Table 6, the initial diagram is presented with the objective, criteria and alternatives:

Figure 2: Hierarchical structure of the AHP



Own elaboration based on Saaty (1980)

For the structuring of the criteria matrix, the architect Ernesto Mock, an expert appraiser who lives in Panama City, was consulted, and the following result was obtained:

Table 7: Matrix of criteria comparisons

	Construction area	**Parking place**	**Age**	**Quality**	**Floor**	**Distance VP**
Construction area	1	5	3	3	4	3
Parking place	1/5	1	1/4	1/5	1/5	1/5
Age	1/3	4	1	2	3	3
Quality	1/3	5	1/2	1	3	3
Floor	1/4	5	1/3	1/3	1	2
Distance VP.	1/3	5	1/3	1/3	1/2	1
Sum	2,45	25,00	5,42	6,87	11,70	12,20

Own elaboration based on Camacaro Mock (2021))

We proceed to normalize the matrix by sum, calculate the eigenvector, the total vector and the λ_{max}:

Table 8: Matrix normalization, eigenvector, total vector and λ_{max}

	Normalization						**Eigen vector**	**Total Vector**	**TV/VT**
Ac	0,4082	0,2000	0,5538	0,4369	0,3419	0,2459	**0,3644**	2,46	6,74
Pe	0,0816	0,0400	0,0462	0,0291	0,0171	0,0164	**0,0384**	0,24	6,28
E	0,1361	0,1600	0,1846	0,2913	0,2564	0,2459	**0,2124**	1,46	6,89
C	0,1361	0,2000	0,0923	0,1456	0,2564	0,2459	**0,1794**	1,22	6,77
P	0,1020	0,2000	0,0615	0,0485	0,0855	0,1639	**0,1103**	0,71	6,48
DPV	0,1361	0,2000	0,0615	0,0485	0,0427	0,0820	**0,0951**	0,59	6,25
								λ_{max}	6,57

Own elaboration based on Camacaro Mock (2021)

The Consistency Index is calculated ***C.I.***:

$$C.I. = \frac{(\lambda_{max}) - n}{(n) - 1} = \frac{6{,}57 - 6}{6 - 1} = 0{,}11$$

We proceed to calculate the Consistency Ratio ***C.R.***, previously the Random Index (***R.I.***) is determined, entering in Table 3 for *n*=6; ***R.I.*** is equal to 1.24:

$$C.R = \frac{C.I.}{R.I.} = \frac{0{,}11}{1{,}24} = 0{,}09 < 0{,}10 \qquad \text{Complies with Table 4}$$

Since all the variables are quantitative (direct and inverse)[6], the eigenvectors for each alternative were determined using the normalization by the sum in this table:

Table 9: Eigenvectors of the alternatives, criteria and general weighting vector

Alternative	***Ac***	***Pe***	***E***	***C***	***Pi***	***DP V***	**Criteria vector**	**General vector**	**Ranking**
PH Arenas Park	0,31	0,14	0,20	0,17	0,25	0,25	0,36	**0,2423**	3
PH Vista Marina Tower	0,20	0,29	0,12	0,05	0,17	0,19	0,04	**0,1554**	4
PH Coral Tower	0,26	0,29	0,20	0,61	0,28	0,37	0,21	**0,3213**	1
PH Miradores Residences	0,23	0,29	0,49	0,17	0,30	0,19	0,18	**0,2810**	2
							0,11		
							0,10		

Own elaboration based on Camacaro & Mock (2021)

According to the results of the AHP application for the selection of the best investment alternative, PH Coral Tower with a total price of USD 200,000 represents the best option.

3.3 From the Multiple Linear Regression Model and the AHP

The calculations are now made by replacing the values of the scale in Saaty's table with the standardized coefficients (Figure 2)[7] of the MLRM variables, according to the following table:

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[6] Except for the Quality variable, which despite having a scale of values for the MLRM analysis, Architect Ernesto Mock again provided his opinion on the values of the pairwise comparison matrix of the alternatives based on the Quality variable. For this study, it was not considered pertinent to survey other professionals, since the methodological proposal of this article does not require the opinions of third parties because the econometric model allows inferring the behavior of the variables and their correspondence with the real estate market prices.

[7] Tests were made with the variations of the adjusted R^2 adjusted, the elasticities of the mean value of the explained variable, the correlations between the independent variables, as well as the base factors associated with the variables in the estimating equation, but the results with the t^* are more robust because they are generated directly from the model.

Table 10: Criteria Comparison Matrix (t*)

Variable	*t**	*Ac*	*Pe*	*E*	*C*	*P*	*DPV*
Ac	11,9	1	5,38	4,33	6,69	2,80	3,56
Pe	2,21	0,16	1	0,80	1,24	0,52	0,66
E	2,75	0,23	1,247	1	1,55	0,65	0,82
C	1,78	0,15	0,81	0,65	1	0,42	0,53
P	4,25	0,36	1,92	1,56	2,39	1	1,27
DPV	3,34	0,28	1,51	1,21	1,88	0,79	1
	Sum	2,20	11,87	9,54	14,74	6,17	7,85

Own elaboration based on Camacaro & Mock (2021)

The matrix is normalized by the sum. We calculate the eigenvector, the total vector and the λ_{max}:

Table 11: Matrix normalization, eigenvector, total vector and λ_{max}.

	N*ormalization*						***Eigen vector***	***Total vector***	**TV/EV**
Ac	0,45	0,45	0,45	0,45	0,45	0,45	0,45	2,72	6,00
Pe	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,51	6,00
E	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,63	6,00
C	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,41	6,00
P	0,16	0,16	0,16	0,16	0,16	0,16	0,16	0,97	6,00
DPV	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,76	6,00
								λmax	6,00

Own elaboration based on Camacaro & Mock (2021)

The Consistency Index is calculated ***C.I.***:

$$\boldsymbol{C.I.} = \frac{(\lambda_{max}) - n}{(n) - 1} = \frac{6-6}{6-1} = 0{,}00$$

We proceed to calculate the Consistency Ratio ***C.R.***, previously the Random Index (***R.I.***) is determined, entering in Table 3 for *n*=6; ***R.I.*** is equal to 1.24:

$$\boldsymbol{C.R} = \frac{\boldsymbol{C.I.}}{\boldsymbol{R.I.}} = \frac{0{,}00}{1{,}24} = 0{,}00 < 0{,}10$$ Complies with Table 4[8]

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8 When starting from a proportional mathematical relationship between values, in this case of standardized coefficients of the variables, it is the same as normalizing by the sum without making the pairwise comparison matrix, in both cases, the R.C. will always be equal to zero.

Since all the variables are quantitative (direct and inverse), the eigenvectors for each alternative were determined using the normalization by the sum summarized in the following table:

Table 12: Eigenvectors of the alternatives, criteria and general weighting vector general

Alternative	*Ac*	*Pe*	*E*	*C*	*Pi*	*DP V*	Criteria vector	General vector	Ranking
PH Arenas Park	0,31	0,14	0,20	0,25	0,25	0,25	0,45	0,26	3
PH Vista Marina Tower	0,20	0,29	0,12	0,13	0,17	0,19	0,08	0,19	4
PH Coral Tower	0,26	0,29	0,20	0,38	0,28	0,37	0,10	0,28	1
PH Miradores Residences	0,23	0,29	0,49	0,25	0,30	0,19	0,07	0,27	2
							0,16		
							0,13		

Own elaboration based on Camacaro & Mock (2021)

According to the results of the AHP application for the selection of the best investment alternative, PH Coral Tower with a total price of USD 200,000 represents the best option.
In order to verify the results obtained with the two approaches, a sensitivity analysis based on the "Gauss Factor" suggested by Dos Santos *et al* (2021) was performed with the same data, obtaining the following results:

Table 13: Eigenvectors of the alternatives, criteria and overall weighting vector

	Saaty's Table	Gauss Factor	Statistical (t*)
PH Arenas Park	0,24	0,23	0,26
PH Vista Marina Tower	0,16	0,17	0,19
PH Coral Tower	0,32	0,29	0,28
PH Miradores Residences	0,28	0,32	0,27
Mean	0,25	0,25	0,25
Standard deviation	0,07	0,07	0,04
Coefficient of variation (C.V.)	28%	26%	17%

Own elaboration based on Camacaro (2022)

In this case, when the Gaussian proposal[9] is applied, PH Miradores Residences is chosen as opposed to the solutions with AHP and MLRM+AHP (PH Coral Tower). The value of PH Coral Tower (Saaty) is equal to the value of PH Miradores Residences (Gaussian), maintaining the difference (0.04) for the properties when changing the calculation method. While the difference between these properties is much smaller when using the t* statistics (0.01).
It is also observed that there is a greater variation between the values of the alternatives in the general weighting vector when using the Saaty's Table, *C.V.* equal to 28%, reducing this variation when using the Gauss Factor, *C.V.* equal to 28%, and even more when using the t* statistics, *C.V.* equal to 17%.

[9] It should be noted that in this analysis the total price was considered as a quantitative variable. The normalization process yielded a high incidence of age in the weighting results. This procedure depends on the quantitative variables and not on surveys for the decision matrix.

4. DISCUSSION

When MADMs are used with an empirical foundation, decision makers (DMs) are very likely to optimize the selection of the best alternative for their investment, business or venture. However, the solution provided by a multi-criteria method alone does not necessarily represent the final decision.

Since the appearance of the popular multi-criteria AHP method, the fundamental scale of the Saaty Table has been widely investigated, with mixed opinions in the scientific community.

The suggested methodology for the replacement of the pairwise comparison in the criteria and alternatives matrices by numerical matrices based on relationships between MLRM test statistics and the use of quantitative variables, requires the real estate consultant to have full knowledge of the behavior of real estate prices and the variables involved in their formation.

The figure of the real estate appraiser is key to analyze the databases, and with the use of scientific methodology generate the econometric models to extract the test statistics that empirically support the solutions when the AHP is used.

Currently, there are several computer programs, such as Expert Choice and Super Decisions, which facilitate access to mathematical calculations that allow consolidating the knowledge and development of the AHP multi-criteria method. There are also several technological tools that allow, in this era of Big Data, the processing of data to provide solutions when it is necessary to make a decision using multiple criteria.

5. CONCLUSION

The use of the matrix of the relationships between the t^* statistics associated with the MLRM variables replaces the expert judgments in the surveys based on the Saaty Table, allowing decisions to be made with a sufficient degree of objectivity, thus mitigating the subjectivity in the heuristic solution provided by the AHP.

For the analysis of real estate offers, the t^* Criterion represents a valid alternative and serves to support the work of real estate professionals, with the advice of expert appraisers, when using the AHP to optimize the marketing of real estate products and satisfy the requirements of potential buyers for the best investment decision.

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