

# Transforming AHP Group Decisions into Fuzzy Priorities – A New Methodology to Aggregate Individual Priorities



**Oliver Meixner\*, Sonja Hackl, Rainer Haas – Institute of Marketing & Innovation,  
BOKU University of Natural Resources and Life Sciences, Vienna / Austria**

# Problem description

- ① Group decision making and AHP
- ① number of well-informed decision makers ( $dm$ )
- ① Aggregation of evaluations →  
Consensus? AIJ vs. AIP? (Forman and Peniwati, 1998)
- ① individual priorities that were approximated  
by applying the Analytic Hierarchy Process (AHP)  
AIP:  $dm_1, dm_2, dm_2, \dots, dm_k$
- Crisp numbers of priorities of  $dm_1, dm_2, dm_2, \dots, dm_n$
- transformed into fuzzy numbers



*Research Question: It is possible to aggregate individual AHP priorities in a non-consensus decision situation by transferring crisp AHP priorities into fuzzy AHP priorities?*

# Group decision making and the AHP



- ① cover the whole spectrum of answers more or less comparable results (consistent evaluations)
- ① new approach of covering different opinions of decision making by transforming individual priorities into fuzzy numbers (Zadeh, 1965)
- ① Differs from, e.g., Aggregation of Individual Preference Structures (AIPS) (Escobar and Moreno-Jiménez, 2007)
- ① individual objectives of different actors are incorporated
- ① approximation of an aggregated preference structure
  
- ① Applications of FAHP: e.g., natural resources management (Srdjevic and Medeiros, 2008), industrial applications (Ling and Wu, 2004), computer integrated manufacturing systems (Bozdog, Kahraman and Ruan, 2003), project management and team formation (Wi *et al.*, 2009), ...

## AHP → FAHP

- ① Cover the whole range of evaluations in a group decision situation
- ① AIP → FAHP
- ① triangular fuzzy number  $\tilde{M} = (l, m, u)$  (Chang, 1996, 650)
- ① membership function  $\mu(x)$  reaching from 0 to 1

$$\mu(x) = \begin{cases} \frac{x - l}{m - l}, & x \in [l, m] \\ \frac{u - x}{u - m}, & x \in [m, u] \\ 0, & \text{otherwise} \end{cases}$$

- ①  $\tilde{M}_1$  is covering a whole spectrum of possible outcomes

- ③  $K$  decision makers evaluating an AHP decision hierarchy containing  $I$  elements
- ③ a priority vector  $W_{ik}, i = 1 \dots I, k = 1 \dots K$ .
- ③ aggregate the individual crisp priorities  $W_{ik}$  into one fuzzy priority vector

$$\tilde{W}_{ik} = \{\min(W_{ik}), \bar{W}_{ik}, \max(W_{ik})\}$$

- ③ basic operations of triangular fuzzy numbers  $\tilde{M}_1 = (l, m, u)$  number  
 $\tilde{M}_2 = (l, m, u)$

$$\tilde{M}_1 \oplus \tilde{M}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$

$$\tilde{M}_1 \otimes \tilde{M}_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2)$$

$$\tilde{M}_1^{-1} \approx \frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}$$

# Numerical example

- ① Panel of 8 experts
- ① Evaluated the sustainability of palm oil



# Numerical example

- ① Panel of 8 experts
- ① Evaluated the sustainability of palm oil
- ① based on an AHP hierarchy
- ① evaluation of criteria by pairwise comparisons
- ① evaluation of the alternatives by quantitative information

<b>1 Ecological sustainability</b>	<b>2 Economic sustainability</b>	<b>2 Social sustainability</b>
1.1 Climate change	2.1 Productivity	3.1 Basic needs
1.2 Air, water, soil quality	2.2 Profitability	3.2 Empowerment
1.3 Waste	2.3 Relative poorness	
1.4 Biodiversity	2.4 Inclusion	
1.5 Use of resources		

Conventional palm oil	RSPO-certified palm oil	Rapeseed oil
-----------------------	-------------------------	--------------

Alternatives Criteria		PO	PO $w_i$	RPO	$w_i$	RO	$w_i$
1 Ecological sust.							
1.1 Climate change	t CO <sub>2</sub> -equivalents	5.34	0.20	3.41	0.31	2.22	0.48
1.2 Air, water, soil quality	Acidification kg SO <sub>2</sub> /t oil <sup>a</sup>	14.8		10.3		20.2	
	Eutrophication NO <sub>3</sub> /t oil <sup>a</sup>	124	(0.30,0.31,0.32)	86	(0.43,0.44,0.45)	140	(0.23,0.25,0.27)
1.3 Waste		<sup>b</sup>	0.14	<sup>b</sup>	0.43	<sup>b</sup>	0.43
1.4 Biodiversity	PDF (potentially disappeared fraction) / m <sup>2</sup> / year	2.04	0.17	1.62	0.33	7.13	0.50
1.5 Use of resources	Megajoul MJ / ha	2.11	0.398	2.11	0.398	4.116	0.204
2 Economic sust.							
2.1 Productivity	corp yield t/ha	3.75	0.37	5	0.49	1.5	0.15
2.2 Profitability	USD/t	700	0.29	800	0.33	900	0.38
2.3 Relative poorness	gross income of local farm workers (USD)	352	0.24	460	0.65	1801	0.11
2.4 Inclusion	employment and income opportunities for local population <sup>b</sup>	<sup>b</sup>	0.17	<sup>b</sup>	0.33	<sup>b</sup>	0.50
3 Social sust.							
3.1 Basic needs	Access to water, housing, sanitary facilities <sup>a</sup>	<sup>b</sup>	0.17	<sup>b</sup>	0.33	<sup>b</sup>	0.50
3.2 Empowerment	Information, knowledge, fair partnership <sup>a</sup>	<sup>b</sup>	0.20	<sup>b</sup>	0.20	<sup>b</sup>	0.60

Alternatives *a*: Conventional palm oil (PO), RSPO-certified palm oil (RPO), Rapeseed oil (RO)

<sup>a</sup> Two indicators available, the bandwidth was interpreted as fuzzy numbers.

<sup>b</sup> As results from literature are ambiguous or not completely comparable, simplified ratings were used.



Alternatives	PO	PO w <sub>i</sub>	RPO	w <sub>i</sub>	RO	w <sub>i</sub>	
<b>Alternatives</b>			<b>PO</b>	<b>PO w<sub>i</sub></b>			
<b>Criteria</b>							
1 Ecological sust.						0.48	
1.1 Climate change	t CO <sub>2</sub> -equivalents		5.34	0.20		(0.25,0.27)	
1.2 Air, water, soil quality	Acidification kg SO <sub>2</sub> /t oil <sup>a</sup>		14.8			0.43	
	Eutrophication NO <sub>3</sub> /t oil <sup>a</sup>		124	(0.30,0.31,0.32)		0.50	
1.3 Waste			b	0.14		0.204	
1.4 Biodiversity	PDF (potentially disappeared fraction) / m <sup>2</sup> / year		2.04	0.17		0.15	
						0.38	
1.5 Use of resources	Megajoul MJ / ha		2.11	0.398		0.11	
						0.50	
3 Social sust.							
3.1 Basic needs	Access to water, housing, sanitary facilities <sup>a</sup>	b	0.17	b	0.33	b	0.50
3.2 Empowerment	Information, knowledge, fair partnership <sup>a</sup>	b	0.20	b	0.20	b	0.60

Alternatives *a*: Conventional palm oil (PO), RSPO-certified palm oil (RPO), Rapeseed oil (RO)

<sup>a</sup> Two indicators available, the bandwidth was interpreted as fuzzy numbers.

<sup>b</sup> As results from literature are ambiguous or not completely comparable, simplified ratings were used.

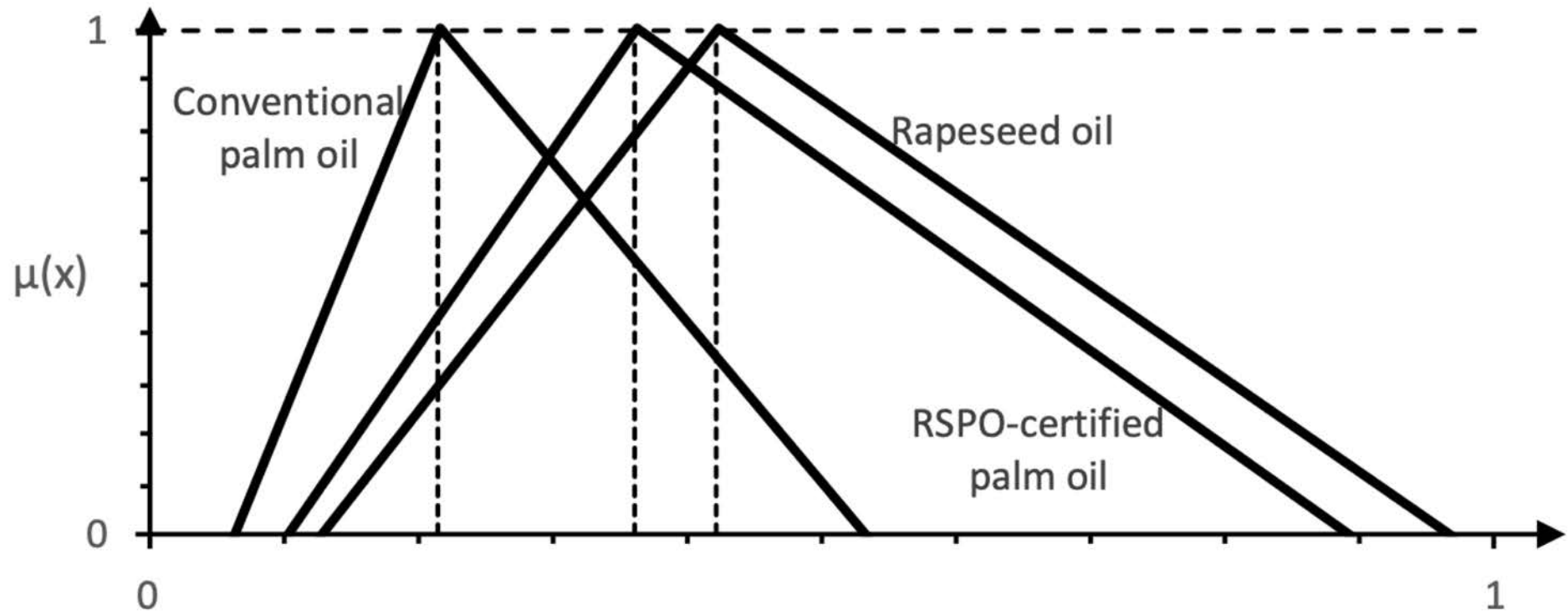
# Results

Experts	E1	E2	E3	E4	E5	E6	E7	E8	min	mean	max	$\tilde{W}_i$
Criteria												
1 Ecological sust.	0.47	0.41	0.33	0.59	0.43	0.43	0.69	0.22	0.22	0.45	0.69	(0.22,0.45,0.69)
1.1 Climate change	0.20	0.41	0.20	0.25	0.33	0.32	0.46	0.16	0.16	0.29	0.46	(0.16,0.29,0.46)
1.2 Air, water, soil quality	0.20	0.12	0.20	0.14	0.12	0.23	0.21	0.19	0.12	0.18	0.23	(0.12,0.18,0.23)
1.3 Waste	0.20	0.07	0.20	0.14	0.04	0.04	0.06	0.19	0.04	0.12	0.20	(0.04,0.12,0.20)
1.4 Biodiversity	0.20	0.26	0.20	0.33	0.38	0.23	0.15	0.26	0.15	0.25	0.38	(0.15,0.26,0.38)
1.5 Use of resources	0.20	0.14	0.20	0.14	0.13	0.19	0.12	0.19	0.12	0.16	0.20	(0.12,0.16,0.20)
2 Economic sust.	0.05	0.26	0.33	0.08	0.14	0.14	0.09	0.46	0.05	0.19	0.46	(0.05,0.19,0.46)
2.1 Productivity	0.07	0.07	0.30	0.08	0.17	0.09	0.15	0.24	0.07	0.15	0.30	(0.07,0.15,0.30)
2.2 Profitability	0.04	0.07	0.10	0.04	0.17	0.10	0.09	0.33	0.04	0.12	0.33	(0.04,0.12,0.33)
2.3 Relative poorness	0.44	0.44	0.30	0.44	0.50	0.43	0.35	0.24	0.24	0.39	0.50	(0.24,0.39,0.50)
2.4 Inclusion	0.44	0.42	0.30	0.44	0.17	0.38	0.41	0.19	0.17	0.34	0.44	(0.17,0.34,0.44)
3 Social sust.	0.47	0.33	0.33	0.33	0.43	0.43	0.22	0.32	0.22	0.36	0.47	(0.22,0.36,0.47)
3.1 Basic needs	0.80	0.75	0.75	0.50	0.88	0.50	0.75	0.50	0.50	0.68	0.88	(0.50,0.68,0.88)
3.2 Empowerment	0.20	0.25	0.25	0.50	0.13	0.50	0.25	0.50	0.13	0.32	0.50	(0.13,0.32,0.50)

(All evaluations were consistent with CR < 0.1)



# Results



## Conclusions & Limitations

- ① case presented: huge range of the fuzzy priorities based on different positions of  $dm$
- ① group decision by far not homogeneous
- ① the assessment of sustainability significantly → individual position of the decision makers & their associated organization
- ① good chance that the alternative RSPO evaluated better
- ① using AIJ, rapeseed oil → the most sustainable alternative
  
- ① goal is to visualize heterogeneity  
→ approach is beneficial
- ① goal is to make an actual decision → AIJ



# Thank you for your attention!

Oliver Meixner\*, Sonja Hackl, Rainer Haas

Institute of Marketing & Innovation

University of Natural Resource and Life Sciences, Vienna

