CRISP JUDGEMENT SCALE-BASED SENSITIVITY ANALYSIS FOR PROVIDING RELIABLE QUALITATIVE AHP/ANP USE OUTCOMES

ABSTRACT

AHP/ANP comprises qualitative technique that is capable of dealing with intangible issues. The application of standard crisp linear judgement 0-9 scale provides user with results that are of a rather quantitative nature, however. This is why, among other things, other measures (fuzzy numbers, gray numbers, rough sets etc.) are applied to express influence of intangible issues in AHP/ANP technique to obtain qualitative analysis results, instead. The application of non-crisp notions for the expression of intangibility seems rather doubtful in light of opinion of the creator of the technique. This is why an effort is made in the paper to provide necessary means to facilitate obtaining qualitative AHP/ANP application outcomes while using crisp input data. Concurrent application of different alternative crisp judgement scales is proposed with this regard. The results of their sample application are also discussed in the paper.

Keywords: AHP/ANP, analysis, qualitative outcomes, crisp judgement scale, sensitivity analysis.

1. Introduction

Standard crisp-based AHP/ANP technique application results in quantitative outcomes. This fact seems to contradict with a demonstrated qualitative nature of the technique. The application of different non-crisp measures has been proposed to deal with the above mentioned contradiction. The application of such measures is rather inacceptable because their non-crisp nature contradicts with fundamental AHP rules, according to technique inventor's point of view. This is why a specific application of crisp measures is proposed to make obtaining qualitative AHP/ANP application results possible. The application of different alternative judgement scales is discussed in the paper in this regard.

2. Literature Review

Different non-crisp measures have been proposed to provide adequate means for making AHP/ANP analysis outcomes compatible with qualitative nature of the technique. Both fuzzy sets (Shi et al., 2023), grey numbers (Zhang et al., 2023), rough sets (Fan et al., 2022) etc. They are, however, rather unreliable due to a number of serious concerns (Saaty, 2006). Note there have also appeared some initial proposals for advanced qualitative analysis of AHP/ANP application (Ginda, & Szyba, 2020). They are nevertheless usually based on some specific, and rather subjective assumptions.
Therefore, that there is an urgent need to elaborate a reliable and comprehensive way for providing really qualitative and objective AHP/ANP analysis results.

3. Hypotheses/Objectives

The main objective of the paper deals with the definition of a comprehensive and reliable approach for providing AHP/ANP analysis results that are compatible with its qualitative character.

4. Research Design/Methodology

A concurrent application of available alternative crisp judgement scales is proposed for the approach. The application of diverse judgement scales results in specific qualitative data representation – intervals of final priorities for decision making alternatives.
There are three kinds of alternative judgement scale types which differ, amongst others, in value expressing the highest scale levels:

- a real number less than 9 (low point judgement scales),
- 9 (regular scales),
- and a real number greater than 9 (high point scales).

The available judgement scales also differ a lot in character of a function that approximates judgment values for the consecutive scale levels $o = 1, 2 \ldots 9$. We can notice linear, concave as well as convex functions, here.
The following AHP/ANP judgement scale alternatives are available:

1. Low point scale alternatives L:

   L1: Saaty's root scale: $a_{ij} = c\sqrt{o}$, where: c =1.

   L2: Root scale (Harker & Vargas, 1987): $a_{ij} = \sqrt[c]{o}$, where: $c = \sqrt{2}$.

   L3: Logarithmic scale (Ishizaka et al., 2006): $a_{ij} = \log_c o + 1$, where: $c = e$.

   L4: Multiplicative scale (Dodd et al., 1992): $a_{ij} = \exp\left[\tanh^{-1}\left(\frac{o-1}{H-1}\right)\right]$, where: $H = 1 + 6 \cdot \sqrt{2}$.

   L5: Finan & Hurley (1997) scale: $a_{ij} = (1 + s)^{\frac{0-1}{2}}$, where: $s = [0.2, 0.732)$.

2. Regular scales N:

   N1: Standard linear Saaty's scale: $a_{ij} = c \cdot o$, where: c = 1.

   N2: Linear reciprocal scale (Ma & Zheng, 1991): $a_{ij} = \frac{9}{10-o}$.

   N3: Balanced scale (Salo & Hämäläinen, 1997): $a_{ij} = \frac{0,5+(o-1)\cdot s}{0,5-(o-1)\cdot s}$,
   where: s = 0,05 .

   N4: Equidistant scale (Rašković et al., 2008): $a_{ij} = v_o$, where:
   $v_o = v_{o-1} \cdot s$ for $o > 1, v_1 = 1, s = \sqrt[8]{9}$.

   N5: Finan and Hurley (1997) scale: $a_{ij} = (1 + s)^{\frac{0-1}{2}}$, where: $s = 0.732$.

3. High point scales H:

   H1: Saaty's quadratic scale: $a_{ij} = c \cdot o^2$, where: c = 1.

   H2: Exponential scale (Harker, & Vargas, 1987): $a_{ij} = o^c$, where: $c = \sqrt{2}$.

H3: Geometric scale no. 1 (Lootsma, 1996; Lootsma, 1997):

$a_{ij} = c^{o-1}$, where: $c = e$.

H4: Geometric scale no. 2 (Lootsma, 1996; Lootsma, 1997):

$a_{ij} = c^{o-1}$, where: $c = \sqrt{2}$.

H5: Finan and Hurley (1997) scale: $a_{ij} = (1+s)^{\frac{o-1}{2}}$, where: $s = (0.732, 1]$.

H6: Geometric scale no. 3 (Légrády et al., 1984; Kok & Lootsma, 1985): $a_{ij} = e^{\frac{o-1}{2}} = \exp\left(\frac{o-1}{2}\right)$.

H7: Multiplicative scale (Dodd et al., 1992):

$a_{ij} = \exp\left[\tanh^{-1}\left(\frac{o-1}{H-1}\right)\right]$, where: $H = 1 + \frac{14}{\sqrt{3}}$.

Note that these different judgement scales provide very different judgement values for the same scale levels. They are hard to compare, therefore. Thus, all the scales are put into a common base by means of judgement standardisation. The standardisation is obtained by means of relating judgements pertaining to different scale levels to actual judgement value corresponding to the highest scale level (max o = 9) of a given judgement scale:

$$a'_{ij} = \frac{a_{ij}}{\max_{o}\{a_{ij}\}} . \quad (1)$$

The comparison of different judgement scale standardised alternatives is presented in Fig.1 (low point scales), Fig.2 (regular scales), and Fig.3 (high point scales).

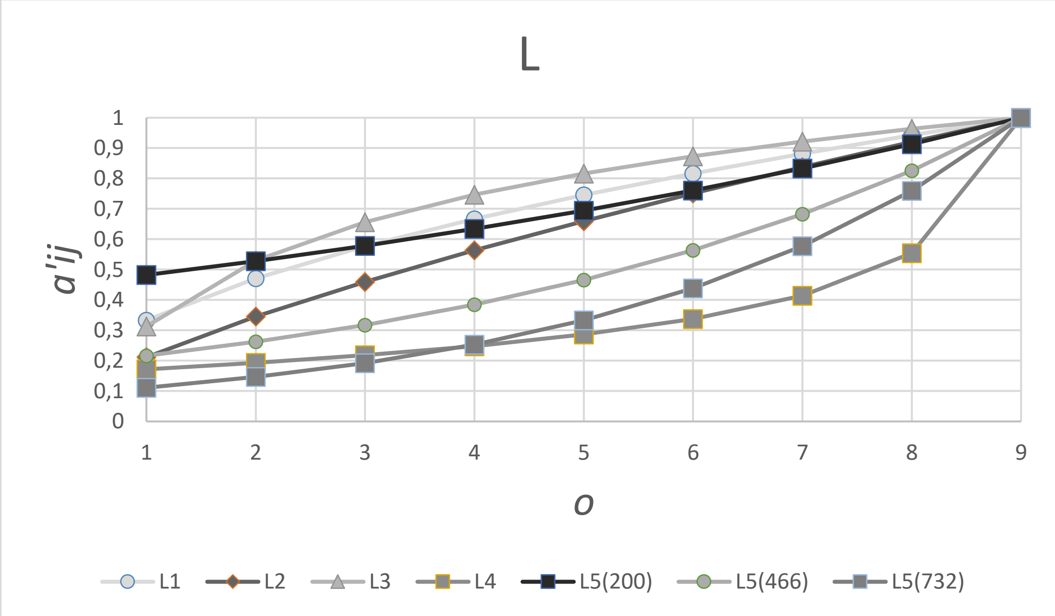

Figure 1. The comparison of standardised low point judgement scales

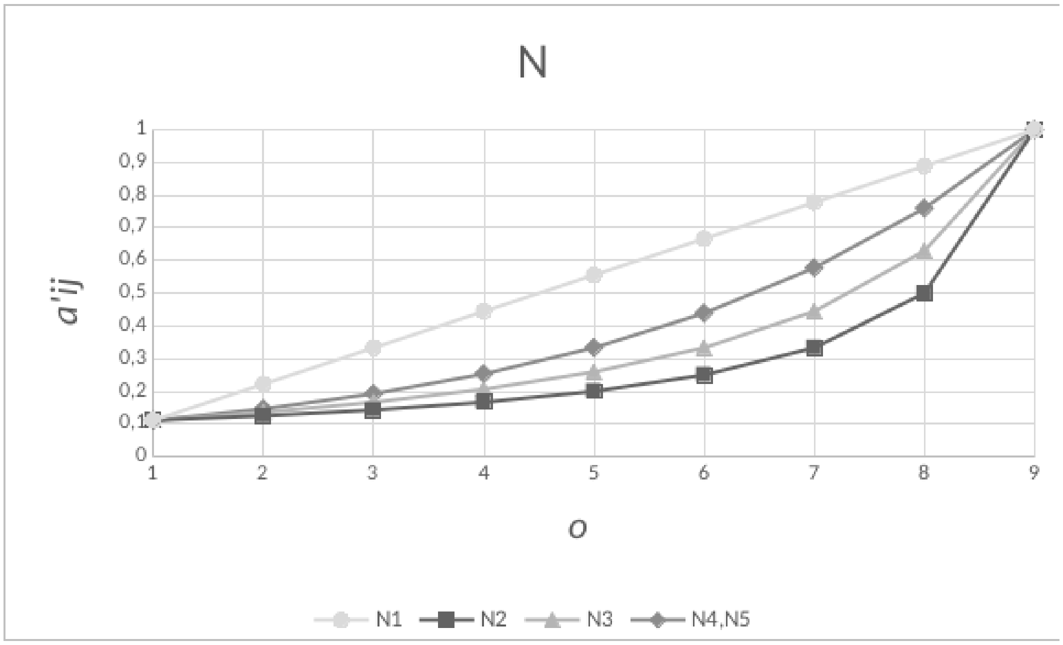

Figure 2. The comparison of standardised regular judgement scales

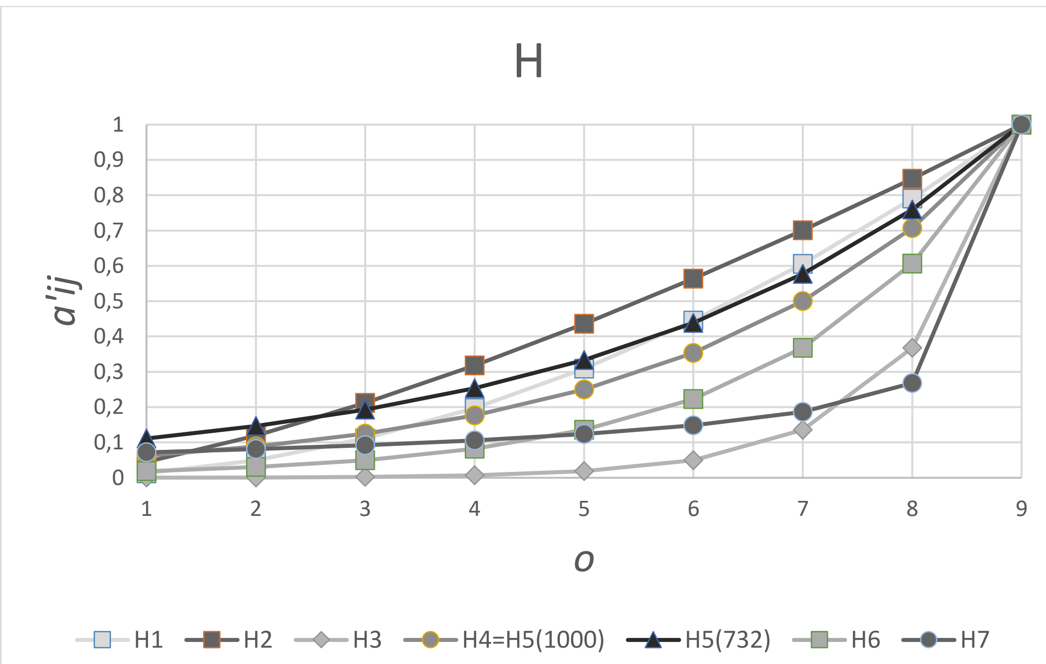

Figure 3. The comparison of standardised high point judgement scales

5. Data/Model Analysis

It is obvious from Figs.1-3 that the majority of approximating functions is convex. A few functions are concave (see: L1, L2, and L3 judgement scales in Fig.1 with this regard). Standard Saaty's scale is the only one linear judgement scale.

Detailed analysis of the flow of functions approximating judgements pertaining to distinct scale levels leads to the conclusion that there are two non-linear functions that are capable of providing highly distinctable judgements across full range of scale levels in cases of concave and convex function families. Concave family o functions is represented well by logarithmic L3 scale by Ishizaka et al. (2006) while geometric H6 scale by Légrády et al., (1984), and Kok & Lootsma (1985) seems to represent the other function family well. The extreme behavior of both above mentioned judgement scales seems to provide necessary means to complement the application of standard Saaty's scale N1. The scale level by scale level changes of judgements for the indicated judgement scales (Fig.4) confirms that.

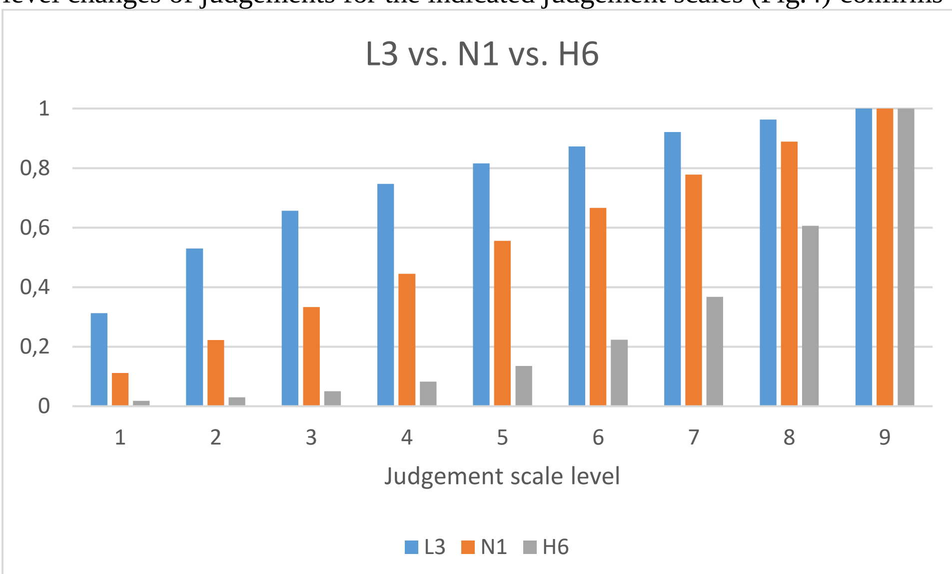

Figure 4. The comparison of standardised high point judgement scales

The initial applicability and usability of indicated scales is tested by means of a sample AHP analysis. A sample dataset of pair-wise judgements is applied with this regard. Relative assessments of the superiority of five distinct alternatives named A, B, C, D, and E is applied with this regard.

The application of standard Saaty's judgement scale for pair-wise comparisons of the alternatives results in the judgements presented in Tab.1.

Table 1. Sample AHP analysis – considered judgements and results (*c.r.* = 0.088)

Alternative	A	B	C	D	E
A	1	1/3	3	6	1/4
B	3	1	5	9	1/2
C	1/3	1/5	1	3	1/3
D	1/6	1/9	1/3	1	1/8
E	4	2	3	8	1

The application of the other two judgement scales is based on the parametric judgements presented in Tab.2.

Table 2. Sample AHP analysis – considered judgements and results (*c.r.* = 0.088)

Alternative	A	B	C	D	E
A	1	1/ a_{ij} (o = 3)	a_{ij} (o = 3)	a_{ij} (o = 6)	1 / a_{ij} (o = 4)
B	a_{ij} (o = 3)	1	a_{ij} (o = 5)	a_{ij} (o = 9)	1/ a_{ij} (o = 2)
C	1/ a_{ij} (o = 3)	1 / a_{ij} (o = 5)	1	a_{ij} (o = 3)	1 / a_{ij} (o = 3)
D	1/ a_{ij} (o = 6)	1 / a_{ij} (o = 9)	1/ a_{ij} (o = 3)	1	1 / a_{ij} (o = 8)
E	a_{ij} (o = 4)	a_{ij} (o = 2)	a_{ij} (o = 3)	a_{ij} (o = 8)	1

The obtained results are gathered in Tab.3 and presented in Fig.5.

Table 3. Sample AHP analysis – considered judgements and results (*c.r.* = 0.088)

Alternative	L3	N1	H6	Min.	Avg.	Max.
A	0.1827	0.1576	0.1282	0.1282	0.1576	0.1827
B	0.2757	0.3185	0.3899	0.2757	0.3185	0.3899
C	0.1274	0.0874	0.0694	0.0694	0.0874	0.1274
D	0.0787	0.0342	0.0129	0.0129	0.0342	0.0787
E	0.3355	0.4023	0.3996	0.3355	0.3996	0.4023
Sum:	1	1	1			

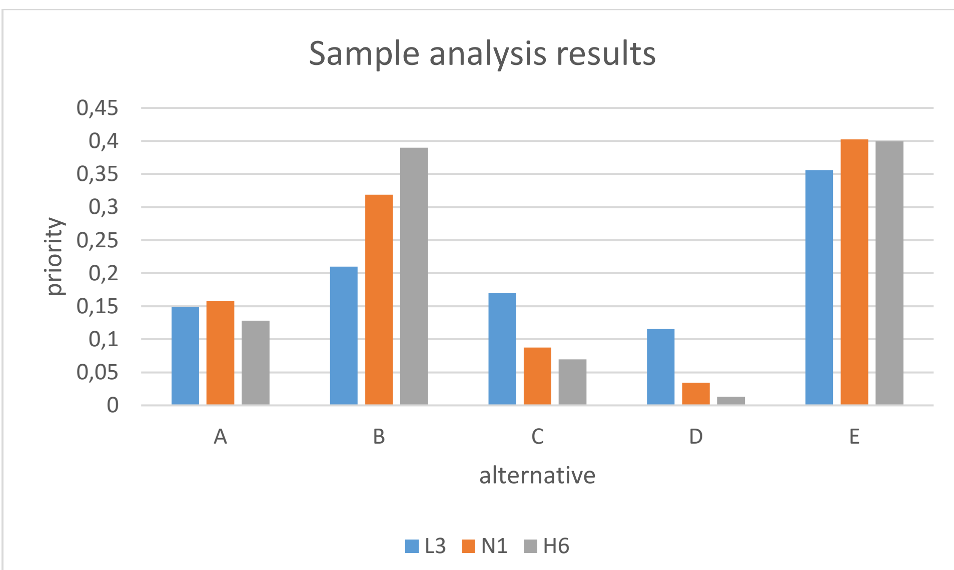

Figure 5. Illustration of sample analysis results

It is evident that the obtained multidimensional sample analysis results allow for more comprehensive and multi-dimensional assessment of alternatives than the results of scalar results provided by the application of a standard Saaty's judgement scale. For example, unlike in the case of a single judgement scale application, the application of multiple

judgement scales provides results that testify for rather close qualitative results for two top alternatives E and B. The similar relation may also appear in the case of two followers, namely A and C alternatives, at least. Moreover, the case of inconsistent advantage of the these alternatives may also yield them incomparable in the sense of their inconsistent behaviour with regard to changes in advantage across different judgement scales.
The obtained results confirm, therefore, the potential of the presented concept for facilitating the interpretation AHP/ANP technique application results in a qualitative manner that is compatible with inherent qualitative character of the technique.

6. Limitations

The current form of presented approach is based on a rather subjective choice of representative alternative judgement scales to make it easier to cope with a problem of a considerable number of available AHP/ANP judgement scales. It seems purposeful, therefore, to undertake further studies that would aim at providing necessary means for a less subjective indication of the most representative alternative judgement scales. Note that the enrichment of identified judgement scale set of is possible, too.

7. Conclusions

AHP/ANP technique seems to suffer from methodological incomparability of provided (quantitative) results and a real qualitative nature of the technique. The outcomes of sample analysis show that concurrent application of diverse alternative crisp judgement scales in AHP/ANP analysis seems to help a lot in mitigation of the incomparability. However, the introduction of further improvements to the approach is also welcome to make it more effective and objective.

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