Proposals for the sequences of pairwise comparisons

ABSTRACT

Pairwise comparison matrices form the basis of the AHP. We provide optimal sets of comparisons of incomplete pairwise comparison matrices (IPCMs) in case of given number of alternatives and given number of comparisons, i.e., the ones that provide the closest weight vectors to the vectors calculated from the complete case. We conduct extensive simulations to compare the different structures of comparisons. It turns out that the regularity and bipartiteness of the representing graphs of IPCMs are the most important properties connected to optimality. Among spanning trees, the star graph is found to be optimal. In many cases the different optimal sets are reachable from each other by adding exactly one comparison to the previous set, resulting in an optimal sequence of pairwise comparisons. Our findings can be utilized not only by the application of a given optimal set of comparisons but also using an optimal sequence in cases when the number of comparisons given by the decision maker is not guaranteed (e.g., in online questionnaires).

Keywords: incomplete pairwise comparison matrix, representing graph, optimal sequence

1. Introduction

One of the corner stones of the AHP is the concept of pairwise comparison matrices (Saaty, 1980). The case of incomplete data, when some elements are missing from the matrix, is popular in today's literature and practice as well. The two most important aspects that affect the results are the number of known comparisons, and their arrangement. Our goal is to provide the sets (arrangements) of comparisons (from all possible ones) that result in the closest weight vectors to the prioritization vectors computed from the complete matrix in case of given number of alternatives (n) and given number of comparisons (e). This way we can estimate the preferences of the decision maker in a problem the best possible way.

2. Literature Review

Szádoczki et al. (2022b) have compared and proposed some special patterns of comparison sets, while other studies also included additional prior information to the analysis (Szádoczki et al., 2022a). However, they only focused on special cases, and compared patterns with different number of comparisons, while we are determining the optimal sets of comparisons among all sets with the same cardinality. Our research also has a strong relationship with previously recommended sequences of pairwise comparisons (see for instance Fedrizzi and Giove (2013)).

3. Hypotheses/Objectives

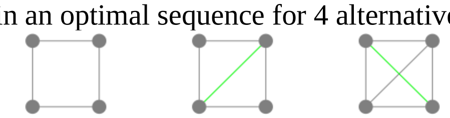
We would like to show the list of optimal comparison sets of IPCMs for given (n,e) pairs. We are particularly interested in whether these optimal sets are reachable from each other creating optimal sequences.

4. Research Design/Methodology

We use extensive numerical simulations with a sample-size of 1 million random matrices to compare the different sets of comparisons for given pairs of (n,e). The gained weight vectors are compared to the vectors calculated from the complete matrix with the help of the Kendall's rank correlation coefficient and the Euclidean distance. The used weight calculation techniques are the eigenvector method and the logarithmic least squares method generalized for IPCMs, while three different inconsistency levels are considered.

5. Data/Model Analysis

It turns out that the results are the same for different inconsistency levels, and in case of both weight calculation techniques. Furthermore, the ordinal and cardinal measures provide the same ranking of the different comparison sets basically in every case. This way we can rank the patterns and determine the optimal one. Some of the representing graphs of optimal sets resulting in an optimal sequence for 4 alternatives can be seen below:



6. Limitations

Our simulations provide evidence on the optimality of the given sets, however, it would be nice to structure our findings in a theorem. Considering the number of alternatives, the computational demand of the simulations is obviously a limitation.

7. Conclusions

We listed the optimal sets of pairwise comparisons, i.e., the ones providing the closest weight vectors to the complete case for given number of alternatives and given number of comparisons from all the possible patterns. It turns out that many of these optimal sets are reachable from each other by adding exactly one comparison to the previous case, thus resulting in optimal sequences. Our results can be useful in problems, when the number of comparisons is not guaranteed, but following an optimal sequence, we can estimate the preferences of the decision maker the best possible way. Interesting future research include the study of larger cases and inclusion relations between the optimal sets of comparisons.

8. Key References

Fedrizzi, M., & Giove, S. (2013). Optimal sequencing in incomplete pairwise comparisons for large dimensional problems. *International Journal of General Systems, 42(4)*, 366–375.

Saaty, T. L. (1980). The Analytic Hierarchy Process. McGraw-Hill, New York.

Szádoczki, Zs., Bozóki, S., Juhász, P., Kadenko, S. V., & Tsyganok, V. (2022a). Incomplete pairwise comparison matrices based on graphs with average degree approximately 3. *Annals of Operations Research*.

Szádoczki, Zs., Bozóki, S., & Tekile, H. A. (2022b). Filling in pattern designs for incomplete pairwise comparison matrices: (Quasi-)regular graphs with minimal diameter. *Omega, 107(C)*:102557.