Prioritized Multi-Commodity Flow Model and Algorithm

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Outline

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Motivation: Disasters

Motivation: Prioritization in Evacuation

Critically injured  → 

Hospitals

Minor injured  →  Primary health

Old age / disable  → 

Care centers

Normal evacuees  →  Safe shelters

Importance: Prioritization

- It is process of deciding the relative importance of objects.
- It emphasize to achieve the objectives and goals.
- It is applicable for large scale disaster management problems.
- Prioritized evacuation is on the basis of case sensitive.
- Multi-commodity model sends evacuees from respective sources to corresponding sinks.

Literature Review

- Ford and Fulkerson [1962]: Maxflow mincut, Quickest, Time expanded, Temporally repeated flow, Multi-commodity flow.
- Minieka [1973]: Maximal lexicographic static flow.
- Megiddo [1874]: Lex-max static flow in single source multi-sink network.
- Hamacher and Tufekci [1987]: Lexicographic min cost flow problem for quickest evacuation of a building.
- Hoppe and Tardos [1994,2000]: Polynomial time algorithm for lexicographic maximum flow problem.
- Fleischer and Skutella [2002,2007]: Multi-commodity flow over time problem.
- Pyakurel and Dempe [2020]: Prioritized max-flow with contraflow approach.

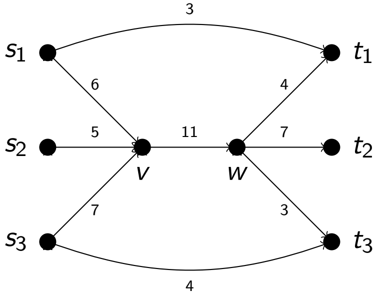
Problem Statement

Problem

Let $\mathcal{N} = (V, A, K, u, d_i, S, D)$ be the given multi-commodity flow network with commodity priority order $\mathbb{P}(i) \prec \mathbb{P}(i+1)\ \forall\, i \in K$. Then the prioritized multi-commodity flow problem is to transship the net flow d_i from s_i to t_i by using priority order without violating the capacity constraints on arcs.

Here, the commodity priority ordering function is $\mathbb{P} : K \to \mathcal{Z}^+$ such that $\mathbb{P}(i) \prec \mathbb{P}(i+1)\ \forall i \in K$, where $\mathbb{P}(i)$ is high priority than $\mathbb{P}(i+1)$.

Multi-commodity Network



$\mathcal{N} = (V, A, K, u, d_i, S, D)$

where,

$\mathcal{N}$ = Network

V = Set of nodes

A = Set of arcs

u = Capacity of arc

S = Set of source node s_i

D = Set of sink node t_i

K = Set of commodities

d_i = Net flow of commodity i

Prioritized Multi-Commodity Flow Model

$$\text{Maximize } x^i$$

s.t.

$$\sum_{e \in A_v^{out}} x_e^i - \sum_{e \in A_v^{in}} x_e^i = \begin{cases} d_i & \text{if } v = s_i \\ -d_i & \text{if } v = t_i \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in K \tag{1}$$

$$0 \le x_e = \sum_{i \in K} x_e^i \le u_e \quad \forall e \in A \tag{2}$$

$$x_e^i = \begin{cases} \nu_P^1 & \text{if } i = 1 \\ \min\{u_e - \sum_{m=1}^{i-1} x_e^m, \ \ \nu_P^i\} & \text{if } i \ge 2 \end{cases} \quad \forall P \in \mathbf{P}_i. \tag{3}$$

where,

$$\nu_P^i = \min\{u_e \mid e \in P\}$$
$$A_v^{in} = \{(w, v) \mid w \in V\}$$
$$A_v^{out} = \{(v, w) \mid w \in V\}$$

Commodity prioritized multi-commodity flow algorithm

Input: Given multi-commodity network $\mathcal{N} = (V, A, K, u, d_i, S, D)$.

1. Prioritize the commodities such that $\mathbb{P}(i) \prec \mathbb{P}(i+1) \;\; \forall i \in K$.
2. Define $S_1 = s_1, \; S_2 = s_1 \cup s_2, \ldots, S_k = \cup_{i=1}^{k} s_i$.
3. Compute lex-max flow using algorithm of Minieka [7] and priority function (3) .

Output: Commodity prioritized multi-commodity flow on $\mathcal{N}$.

Solution Procedure

Theorem

Commodity prioritized algorithm solves commodity prioritized maximum static multi-commodity flow problem efficiently.

Proof.

Feasibility: Steps 1 and 2 are feasible which can be obtained in constant time. The priority function (3) is applied in each intermediate arcs within $O(m)$ times. As lex-max algorithm of Minieka [7] is polynomial time solvable, Step 3 is also feasible.
Optimality: The optimality of algorithm is dominated by the optimality of Step 3 which provides optimal solution for each $S_k = \cup_{i=1}^{k} s_i$. $\square$

Example

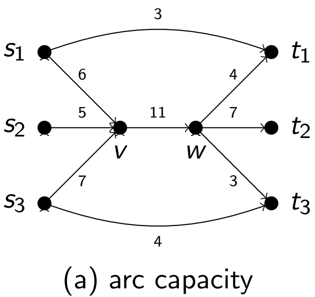


Figure: (a) multi-commodity flow network (b) solution with priority order

Prioritized max-flow for three commodities = 7, 5 and 6 units, respectively.

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Stay Safe !!