ANALYSIS OF THE GAUSSIAN AHP METHOD IN THE LIGHT OF THE PARETO FRONT OBTAINED THROUGH THE MULTI-ATTRIBUTE TRADESPACE EXPLORATION (MATE) METHOD – A CASE STUDY

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ABSTRACT

The MATE method – Multi-Attribute Tradespace Exploration – was proposed by Adam Ross and Nathan Diller while working at NASA. It is based on the Multi Attribute Utility Theory – MAUT –, developed by Ralph Keeney and Howard Raiffa in 1976 and used for eliciting requirements and defining utility functions for a great number of design alternatives, as well as for positioning those alternatives relative to a bidimensional Pareto front. On the other hand, the Gaussian Analytic Hierarchy Process – Gaussian AHP – is an evolution proposed by Dos Santos *et al.* in 2021 for the classic AHP Method, which eliminates the need of pair comparison of attributes for each design alternative and introduces the relationship between standard deviations and mean scores in order to increase the reliability of the generated ranking. In this article the authors use a case study proposed by Adam Ross to confront the Pareto front generated by MATE method with the ranking generated by the Gaussian AHP method and propose a modification of that method.

Keywords: Gaussian AHP, MATE, Pareto front, MBSE.

1. Introduction

MBSE – Model-Based Systems Engineering – emphasizes the application of rigorous, formal analysis methods – usually computational models – throughout the design and development process (BLANCHARD & BLYER, 2016, p. 27). Such a greater rigor is favored by the substitution of paper documents for electronic ones. Here, the word "documents" may refer to requirements specifications, engineering diagrams, risk matrices, configuration controls, change controls, among many others (DE FREITAS OLIVEIRA, 2018 p. 15). One of those instruments is the Multi-Attribute Tradespace Exploration Method proposed by Adam Ross and Nathan Diller while working at NASA (DILLER, 2002 p. 62). It is based on the Multi Attribute Utility Theory – MAUT –, developed by Ralph Keeney and Howard Raiffa in 1976 (NICKEL, 2010 p. 27) and used for eliciting requirements and defining utility functions for a great number of design alternatives. It produces a bidimensional Pareto front, which is used in this article to analyze the results obtained by the Gaussian AHP Method proposed by DOS SANTOS *et al.* (2021).

2. Literature Review

For this article the authors used mainly the master's thesis of Nathan Diller titled "Utilizing Multiple Attribute Tradespace Exploration with Concurrent Design for Creating Aerospace Systems requirements" (DILLER, 2002); the PhD thesis of Adam Ross titled Multi-Attribute Tradespace Exploration with Concurrent Design as a Value-Centric Framework for Space System Architecture and Design" (ROSS, 2003); the master's thesis of Julia Nickel titled "Using Multi-Attribute Tradespace Exploration for the Architecting and Design of Transportation Systems" (NICKEL, 2010); the class materials of the course "Quantitative Methods in Systems Engineering" from MIT, as well as described in the lecture "Concept Design and Tradespace Exploration", from Donna Rhodes and Adam Ross, on 28-October-2014, as part of SEARI – Systems Engineering Research Initiative (SEARI, 2022); and the seminal article from DOS SANTOS *et al* titled "Multicriteria Decision-Making in the Selection of Warships: a New Approach to the AHP Method" (DOS SANTOS, et al., 2021).

3. Hypotheses/Objectives

The authors' objective is to verify the relation of the ranking produced by the Gaussian AHP method with the Pareto front produced by the MATE method for the same attributes and for the same design alternatives and discuss an improvement proposition for the Gaussian AHP method.

4. Research Design/Methodology

The methodology used is built upon a case study authored by Adam Ross and presented in the course "Quantitative Methods in Systems Engineering" from MIT in 2017, as well as described in the lecture "Concept Design and Tradespace Exploration", from Donna H. Rhodes and Adam Ross, on 28-October-2014, as part of SEARI – Systems Engineering Research Initiative, from MIT (SEARI, 2022). The case study consists of eliciting requirements and analyzing design alternatives for a new space tug. After interviews with subject matter experts, the SEARI team reached the final values exhibited in Tables 1 to 5. It is out of the scope of this article to describe those attributes, variables, and formulae –

the reader may refer to SEARI (2022). The authors of this article are only describing the process used by MATE and Gaussian AHP methods.

In Table 1a, the levels on column 4 must be monotonically increasing. Column 5 is the Single Attribute Utility levels – SAU –, respective to each level of the attribute. The SAU levels may be crescent or decrescent but must be monotonic. Note that, for each attribute, the relation between levels and SAU constitutes a utility function: $SAU = f(level)$. For such utility function some points are provided in Table 1a, others will have to be calculated by interpolation.

In the MATE method, MAU – the Multiple Attribute Utility – is calculated using the following expression (ROSS, 2003 p. 57):

$$MAU_m = \sum_{n=1}^{N} SAU_{m,n}\, weight_n \tag{1}$$

Where:

MAU_m is the Multiple Attribute Utility of the design alternative $\boldsymbol{m}$ within $\boldsymbol{M}$ possible design alternatives: $\boldsymbol{m} = \{1, \dots, \boldsymbol{M}\}$.

$SAU_{m,n}$ is the Single Attribute Utility calculated for attribute $\boldsymbol{n}$ of design alternative $\boldsymbol{m}$. It is obtained by a piecewise linear interpolation from the columns 4 and 5 of Table 1a (note that formulae in Table 5 allow for attribute levels not present in Table 1a; thus, some interpolations will be necessary).

$weight_n$ is the weight of attribute $\boldsymbol{n}$, obtained from column 3 of Table 1a.

Note that MAU is a way of consolidating many utility functions of performance in just one utility function. This facilitates the generation of a bidimensional Pareto front (MAU x Total_Cost, as in this instance), rather than generating a higher order Pareto front. There is still some level of subjectiveness in MAU because of the weights of the attributes. Anyway, those weights can be obtained by some trivial ways, such as averaging the weights assigned by different subject matter experts or computing the same attribute more than once, i.e., allowing every subject matter expert to have one dummy attribute added for the same real attribute.

The choice of the design variables, according to the MATE method, is done by a process called Design Value Mapping – DVM –, which results in the variables that have most impact in the attributes of performance and cost. It is out of the scope of this article to enter the DVM process used for this case study: we assume the resulting design variables listed in Table 2.

The MATE method generates $\boldsymbol{M}$ design alternatives by a full factorial combination of the sample levels of the design variables in Table 2. In this case study, even though some design variables are continuous (except for “Propulsion_Type”), there are only 4 samples of the Payload variable, 4 samples of the Propulsion_Type variable, and only 6 samples of the Fuel_Level variable. Thus, in total, there are 96 possible combinations of all 3 variable samples level, each combination being a design alternative. Note: this case study is considered by its authors as “low fidelity” because of the low number of samples for each design variable. Anyway, the authors of this article have developed a software in Python that supports any number of samples for any variable, and a methodology that facilitates the adaptation of the software for other use cases.

For each design alternative, MAU is calculated using expression (1), and the Total_Cost is calculated using the expression in Table 5. At the end, 96 points are obtained, each point being a design alternative. A plot of those points is seen in Figure 1 after normalization of Total_Cost. As may be saw in that figure, the Pareto front has 11 points. Since it is a bidimensional Pareto front, its points are easily identified by inspection.

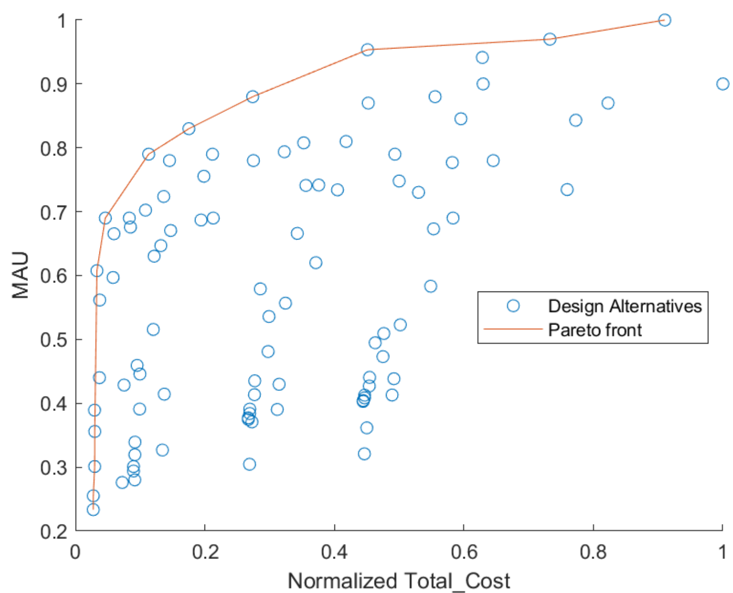

Figure 1: Total_Cost x MAU, and Pareto front

Table 1a: Performance attributes

Attribute	Description	Weights	Levels	SAU
Delta_V	Amount of delta V a Space Tug can impart onto target	0.6	[0, 4.3, 8.3, 12]	[0, 0.7, 0.9, 1]
Capability	0, 1, 2, 3, 4: "none", "low", "med", "high", "extreme" degree of capability	0.3	[0, 1, 2, 3, 4]	[0, 0.3, 0.6, 0.9, 1]
Response_Time	0 = "fast", 1 = "slow"	0.1	[0, 1]	[1, 0]

Table 1b: Performance attributes (continued)

Attribute	Unit	Most desirable value	Least desirable value	More is better?
Delta_V	km/s	12	0	TRUE
Capability	Ordinal	4	0	TRUE
Response_Time	Categorical	0	1	FALSE

Table 1c: Cost attributes

Attribute	Unit	Most desirable value	Least desirable value	Less is better?	Weights	Description
Total_Cost	$ M	0	$4,500.00	TRUE	1	Total project cost

Table 2: Design variables

Variable	Unit	Levels
Payload	Kg	[300, 1000, 3000, 5000]
Propulsion_Type	Dimensionless	["Biprop", "Cryo", "Electric", "Nuclear"]
Fuel_Level	Kg	[100, 300, 1000, 3000, 10000, 30000]

Table 3: Auxiliary variables

Variable	Unit	Levels	Note
Isp	sec	{"Biprop":300, "Cryo":450, "Electric":2200, "Nuclear":1500}	Levels depend on "Propulsion_Type" design variable
Base_Mass	kg	{"Biprop":0, "Cryo":0, "Electric":25, "Nuclear":1000}	Levels depend on "Propulsion_Type" design variable
Mass_Frac	dimensionless	{"Biprop":0.12, "Cryo":0.13, "Electric":0.3, "Nuclear":0.2}	Levels depend on "Propulsion_Type" design variable
Capable	ordinal	{0:0, 300:1, 1000:2, 3000:3, 5000:4}	Levels taken from "Payload" design variable
Fast	categorical	{"Biprop":0, "Cryo":0, "Electric":1, "Nuclear":0}	Levels depend on "Propulsion_Type" design variable

Table 4: Constants

Constant	Unit	Value	Description
BMF	%	1	Bus mass as % payload
Costwet	$k/kg	20	Wet mass cost/kg
Costdry	$k/kg	150	Dry mass cost/kg
Gee	m/s^2	9.8	Gravitational constant

Table 5: Formulae

Result	Unit	Formula
Bus_Mass	kg	Bus_Mass = Payload*BMF + Base_Mass["Propulsion_Type"] + Fuel_Level*Mass_Frac[“Propulsion_Type”]
Dry_Mass	kg	Dry_Mass = Payload + Bus_Mass
Total_Mass	kg	Total_Mass = Dry_Mass + Fuel_Level
Delta_V	km/s	Delta_V = Gee * Isp["Propulsion_Type"] * log(Total_Mass/Dry_Mass)/1000
Capability	Ordinal	Capability = Capable["Payload"]
Response_Time	categorical	Response_Time = Fast["Propulsion_Type"]
Total_Cost	$M	Total_Cost = (Costwet*Total_Mass + Costdry*Dry_Mass)/1000

5. Applying the Gaussian AHP Method to the Design Alternatives

The Gaussian AHP Method is then applied by the authors of this article to the 96 design alternatives generated in the previous step, using as attributes “MAU” and “Total_Cost”. As a result, seven points in the Gaussian AHP ranking falls over (i.e.: coincides with) the Pareto front MAU x Total_Cost, as seen in Figure 2.

Let us remember the definition of Pareto front $P(Y)$ in $\mathbb{R}^2$: let f be a function $f: X \longrightarrow \mathbb{R}^2$ where X is a set of decisions and Y is the feasible set of criterion vectors in $\mathbb{R}^2$ where $Y = \{p \in \mathbb{R}^2 : p = f(x),\ x \in X\}$. It is assumed that the preferred directions of criteria values are known. The fact that a point $p_j \in \mathbb{R}^2$ is preferred to (i.e.: strictly dominates) another point $p_k \in \mathbb{R}^2$ is written as $p_j \succ p_k$. Then:

$$P(Y) = \{\, p_j \in Y : \{\forall p_j \nexists\, p_k \in Y : p_k \succ p_j\} \,\} \qquad (2)$$

In words: $P(Y)$ is the set of points p_j belonging to Y such that there is no point p_k also belonging to Y that is preferred to p_j.

In this case study, a point (design alternative) p_j is preferred to a point p_k if p_j has lower Total_Cost, and greater or equal MAU than point p_k.

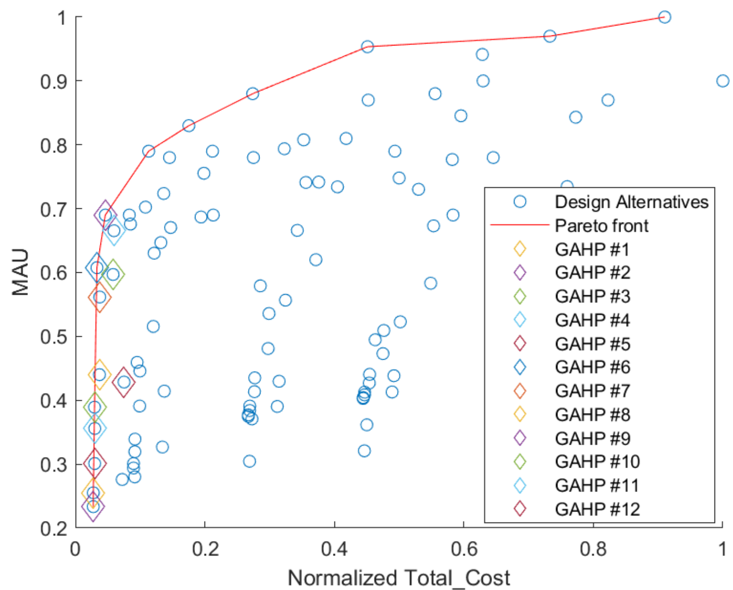

Figure 2: First 12 Gaussian AHP points and Pareto front

From the results shown above, $\boldsymbol{M}$ = 96 design alternatives are generated by the MATE method, constituted of $\boldsymbol{N}$ = 2 attributes from monotonically conflicting utility functions (Total_Cost and MAU), and having $\boldsymbol{P}$ = 11 points in the Pareto front (note that $\boldsymbol{P} << \boldsymbol{M}$). Those design alternatives and attributes are then submitted to the Gaussian AHP method. Interestingly enough, seven points within the range [1st, 12th] of the Gaussian AHP ranking coincides with the Pareto front.

6. Discussion

The Gaussian AHP method produces a ranking that is the inner product of each point (in this case: each attribute of each design alternative) and the transposed normalized standard deviation vector of each attribute, and the result is sorted in decrescent order (DOS SANTOS, et al., 2021 p. 17). The greater the standard deviations, the greater the respective inner product for each row (i.e.: for each design alternative). However, this does not guarantee that the first points with greater standard deviations will appear in the ranking before the ones with lower standard deviations, since the inner product also depends on the point coordinates themselves. Thus, it is not guaranteed that the points located in the extremities of the feasible region will appear in that ranking before the points that are near the center of the feasible region. Since the Pareto front is always in the extremities of the feasible region, it is not guaranteed that its points will appear first in the Gaussian AHP ranking.

To try to improve this situation, a modification of the Gaussian AHP method is proposed for cases with two attributes like this one: rather than using the aforementioned inner product for producing the ranking, here it is proposed to use the bivariate gaussian probability mass function of each point, sorted in crescent order. Since the points in the extremities of the feasible region have lower probability mass function values than points near the center of the feasible region, they will appear first in the ranking. Of course, as before, it is not guaranteed that the first points in the ranking will coincide with the Pareto front. Anyway, for this specific experiment, a better result is obtained: as seen in Figure 3, ten points of the first twelve in the ranking coincides with the Pareto front.

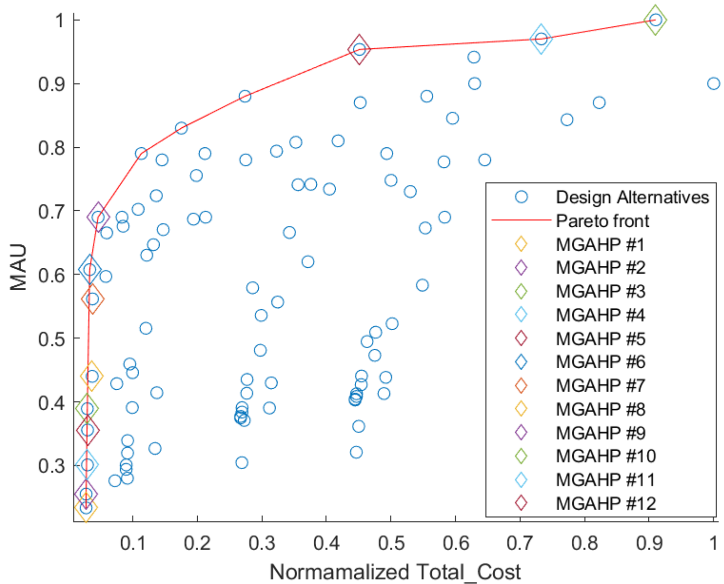

Figure 3: First 12 Modified Gaussian AHP points and Pareto front

7. Suggestions for Future Works

The authors of this article conjecture that, for bidimensional cases like this one, it might be possible to find a closed expression for calculating $\boldsymbol{y}$, the number of points within the first $\boldsymbol{x}$ points of a Gaussian AHP ranking that will coincide with a Pareto front having $\boldsymbol{P}$ points in a set of $\boldsymbol{M}$ points, $\boldsymbol{y} \leq \boldsymbol{P} \leq \boldsymbol{x} \leq \boldsymbol{M}$. In other words, it might exist a function f such that $\boldsymbol{y} = f(\boldsymbol{x}, \boldsymbol{M}, \boldsymbol{P}, \boldsymbol{\Sigma})$: $\boldsymbol{y} \in \mathbb{Z}^{+}, \boldsymbol{x}, \boldsymbol{M}, \boldsymbol{P} \in \mathbb{Z}^{+*}, \boldsymbol{\Sigma} \in \mathbb{R}^{2}$, where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the $\boldsymbol{M}$ points. From this definition it follows that $f(\boldsymbol{M}, \boldsymbol{M}, \boldsymbol{P}, \boldsymbol{\Sigma}) = \boldsymbol{P}$. For this case study: $\boldsymbol{P} = 11, \boldsymbol{x} = 12, \boldsymbol{M} = 96$. Thus, for the Modified Gaussian AHP method and for this specific experiment, $f(12, 96, 11, \boldsymbol{\Sigma}) = f(96, 96, 11, \boldsymbol{\Sigma}) = 10$, and for the original Gaussian AHP method $f(12, 96, 11, \boldsymbol{\Sigma}) = f(96, 96, 11, \boldsymbol{\Sigma}) = 7$.

8. Conclusion

In this article the authors analyze the results of the Gaussian AHP method in the light of a Pareto front generated by the MATE method in a case study for a new space tug. For such case study it is shown that the first twelve design alternatives of the Gaussian AHP ranking are very close to – or coincides with – the Pareto front. It is also proposed a modification of the original Gaussian AHP method to use the bidimensional gaussian probability mass function of a point as the criterium for positioning that point in the ranking; it is shown that – for this case study – the first twelve points of the ranking produced by such modification are closer to the Pareto front than using the original algorithm.

9. References

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