PARTICLE SWARM OPTIMIZATION TO SOLVE INCOMPLETE PAIRWISE COMPARISON

ABSTRACT

According to Saaty (2008) everything we do both consciously and unconsciously is the result of decision making. Analytical Hierarchy Process (AHP) is a method that can assist in making decisions on complex problems by structuring information in a hierarchy. However, the requirement for the AHP method to be completed is a complete pairwise comparison. In fact, you can find incomplete pairwise comparison matrices in AHP. The purpose of this study is to fill in the incomplete pairwise comparison matrix on AHP so that the synthesis process on the AHP method can be completed. To estimate the missing value from pairwise comparison matrices, the research proposed in this article uses the metaheuristic Particle Swarm Optimization Technique (PSO). The results of this study is the proposed algorithm for solving incomplete pairwise comparison using PSO.

Keywords: Analytical Hierarchy Process, Incomplete Pairwise Comparison, Particle Swarm Optimization

1. Introduction

The Analytical Hierarchy Process (AHP) helps in making decisions on complex problems by structuring information in a hierarchy. Incomplete pairwise comparison is a condition in which one or more data in the pairwise comparison matrices is not filled. This condition is often caused by several things, one of which is due to the respondent's lack of understanding of a particular statement in pairwise comparison, causing the respondent to choose not to fill in the pairwise comparison. This condition of incomplete pairwise comparison matrices causes the synthesis process in the AHP method to be unable to be carried out. If incomplete pairwise comparison detection is done manually, it will cause the synthesis process to take a long time. Not only that, if an incomplete pairwise matrices is found, and then the respondent is asked to fill it in again manually. Therefore, it will have the consequence that the respondent must be explained from the beginning of the hierarchical structure, before the respondent can then complete the pairwise comparison matrices. This condition motivates the research presented in this article.

2. Literature Review

Harker (1987) overcomes the incomplete pairwise comparison problem by simplifying the questions. Nishizawa (1997) found the syllogism method to solve the problem of incomplete pairwise comparison matrices. In addition to estimating the missing values from pairwise comparison matrices, some researchers have done this by dividing objects from a level into several subsets (Shen, et al., 1992). Other methods to overcome incomplete pairwise comparison matric such as Neuro Fuzzy (Ichihashi and Tùrksen., 1993); Backpropagation multi-layer perceptron (Hu and Tsai., 2006); Multi-layer Perceptron (MLP) (Gomez-Ruiz, et al., (2010)), graph theory (Bernroider et al., 2010) and Connecting Path Method (CPM)(Chen et al., 2015).

3. Hypotheses/Objectives

The objective of this study is to develop a framework and method based on Particle Swarm Optimization (PSO) to overcome incomplete pairwise comparison matrices.

4. Research Methodology

The research methodology to achieve the objective of the research presented in this paper is shown in Figure 1.

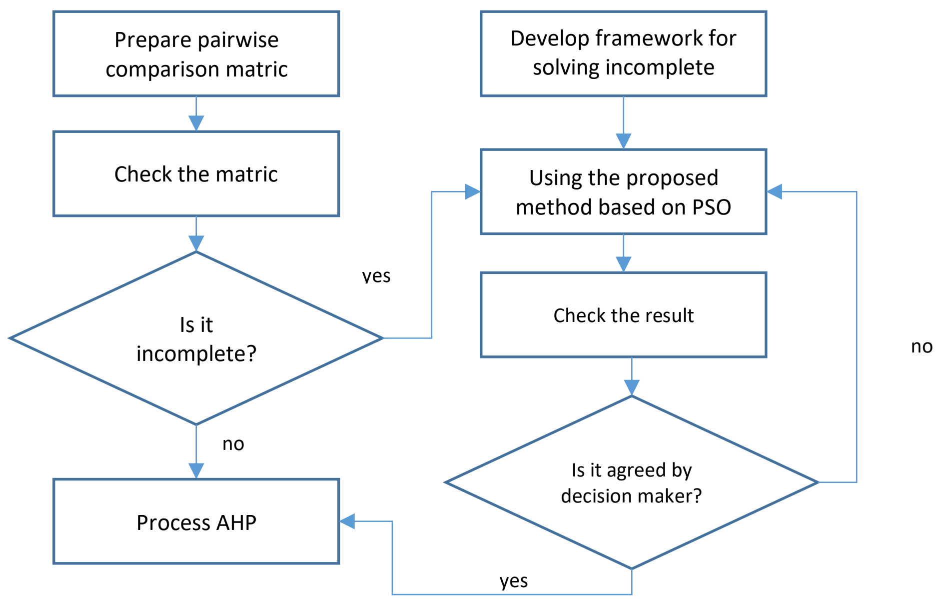

Figure 1. Proposed framework

5. Model Analysis

As shown in Figure 1, the steps in the framework begin with creating the Particle Swarm Optimization algorithm using the C# programming language as follows:

a) Initial population initialization
- Initialize the initial velocity of each particle *Vi (0)* as 0
- Initialize the initial position of each particle *Xi (0)* randomly
- Calculate the fitness where this value reflects the Consistency Ratio (CR)
- Initialize pbest for each particle using the initial position, and gbest using the best pbest

b) Update *velocity*, *pbest* and *gbest* by using the equation described by Marini and Walczak (2015)

c) Update position and calculate fitness value

d) Update *pbest* and *gbest* updates

The algorithm was then tried on real case examples, namely on supplier selection problems. The structure of this problem consists of objectives, criteria and alternatives. There are 3 criteria considered, namely Price, Quality and Variation and 5 alternative suppliers to be evaluated. Incomplete pairwise comparison matric can be seen in Table 1 for incomplete comparison at the criterion level.

Table1. Incomplete Pairwise Comparison at the criterion level

	Price	**Quality**	**Variety**
Price	1	3	4
Quality	1/3	1	-
Variety	1/4	-	1

After the proposed algorithm was applied in that incomplete pairwise comparison matric then the result is presented in Table 2.

Table2. Complete Pairwise Comparison at the criterion level after proposed algorithm is implemented

	Price	**Quality**	**Variety**
Price	1	3	4
Quality	1/3	1	3
Variety	1/4	1/3	1

Based on Table 2 it can be seen that the proposed algorithm can suggest us preference value of the respondent. For the case presented in Table 3, the suggested value is 3 which is the Quality is more important that Variety. The computation of Consistency Ratio (CR) shows us that the value of CR is 0.0639 which is ≤ 0,1. Therefore the pairwise comparison matric is consistent. The time needed to complete the matrix is reasonable which is only < 0.6 second.

6. Limitations

The limitation of the proposed framework is that the algorithm created cannot be embedded in AHP software that is already on the market, so a computer program must be made from the start to solve AHP and also to handle incomplete pairwise comparisons.

7. Conclusions

In the research presented in this article, we propose a framework and method based on Particle Swarm Optimization (PSO) to handle incomplete pairwise comparison matrices. The algorithm tested on the supplier selection problem shows that the proposed algorithm can be used to complete the incomplete pairwise comparison matric with the value of CR is 0.0639 and a completion time of <0.6 seconds.

8. Key References

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