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Power Relations and Group Aggregation in the Multiplicative AHP and SMART

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Abstract

The relative power of the members in a group of decision makers can be incorporated in the Multiplicative AHP (a multiplicative variant of the Analytic Hierarchy Process), and henceforth in SMART (the Simple Multi-Attribute Rating Technique), via so-called power coefficients in the logarithmic least-squares method whereby we analyze the pairwise-comparison matrices. When each decision maker judges every pair of alternatives under each of the criteria respectively, aggregation over the criteria and over the decision makers proceeds via a sequence of geometric-mean calculations which can be carried out in any order.

Key words

Decision analysis, multi-criteria analysis, group decisions, aggregation, criterion weights, power coefficients, logarithmic least squares, geometric means, normalization, rank reversal.

1. Introduction

Although many decisions are made in boards, committees, councils, and not by individual decision makers, the literature on multi-criteria decision analysis pays little attention to the power relations in groups. There is always a power game, however. In national decision-making bodies, each member seems to have a "weight" which is "proportional" to the size of his political affiliation. In international decision-making bodies, the "weights" of the members are somehow "related" to the population size, the military power, or the gross national product of the respective countries. Alternatives which are weakly supported by the "powerful" members have therefore little chance to be accepted by the group, even when a multi-criteria analysis reveals a high degree of support for them under the erroneous assumption that all members would be equal.

For more background information we refer the reader to Galbraith (1983) who defines power as the force that causes the persons subject to it to abandon their own preferences and to accept those of others. In general, there are three kinds of power. Punitive power wins submission by threats, violence, and/or punishment, compensatory power by incentives and/or rewards, and conditioned power by belief, education, and/or indoctrination. The sources of power are personality, property, and organization; in the above kinds of power they are used in various mixtures. The primary source of power today seems to be the disciplined organization such as a state bureaucracy or a modern corporation where administrators and managers exercise their compensatory and conditioned power via their position in the hierarchy. Decision-making teams in such an organization are confronted with the so-called bimodal symmetry: their external power depends on internal discipline. Hence, the need of a compromise which is accepted by all members of the team.

In the present paper we consider an extension of the Multiplicative AHP (a multiplicative variant of the Analytic Hierarchy Process, see Saaty (1980), Saaty and Kearns (1985), and Lootsma (1987, 1993)) designed in order to model power relations in groups. We assign a weight coefficient to each member of the group, and we use these power coefficients in the logarithmic least-squares method whereby we analyze the pairwise-comparison matrices. Although the subsequent calculations remain simple (one only has to solve a linear set of normal equations), it is not clear whether the weighted logarithmic least-squares method yields a plausible equilibrium solution for the power game in the group. And that is the hard question to be answered! We limit ourselves to decision-making processes in a public or private bureaucracy, when the members of the decision-making group have a common interest: they are supposed to present a joint compromise solution to those who asked the group to solve a given problem of choice. Not only the relative power of the group members, but also the criteria and their relative importance may have been prescribed to the group, albeit in vague terms which leave ample space for adjustment and interpretation. Thus, the power game is constrained by what is socially acceptable in the bureaucracy. We do not consider open conflicts in the initial phase when the actors use brute force and when an organisational structure for negotiations and problem solving is still absent. Similarly, we do not try to model the power which a group member may derive from his personality and/or his verbal skills.

First, we analyze the Multiplicative AHP with the weighted logarithmic least-squares method in the special situation that each member of the group judges every pair of alternatives under each of the criteria respectively. This implies that the pairwise comparisons are complete: all cells in the pairwise-comparison matrices have exactly the same number of entries. Moreover, the normal equations associated with the weighted logarithmic least squares have an explicit solution: we can immediately find impact scores for the alternatives by the calculation of geometric row means. Finally, the order of the calculations is immaterial: we may first average over the entries within a cell to obtain a group opinion about the pair of alternatives, and thereafter over a row within the matrix to obtain an impact score for the corresponding alternative, but we may also carry out the calculations in the reverse order. At a higher level, when we aggregate the impact scores (partial scores) of the alternatives under the respective criteria in order to obtain final scores (global scores), we can also compute geometric means in an arbitrary order. The final scores are the same, regardless of whether we average first over the criteria and then over the decision makers, or reversely. Because this is a reasonable requirement for a sequence of aggregations, Barzilai et. al. (1987, 1991, 1992) laid down a set of axioms to guarantee that aggregations may be interchanged in two-level problems with multiple alternatives and criteria; they demonstrated that only geometric means have the desired property. In the present paper we establish the property in three-level problems with multiple alternatives, criteria, and decision makers. To illustrate matters, the original version of this report shows via the notorious example of Belton and Gear (1983) that the Multiplicative AHP does not suffer from rank reversal by the addition of copies to a set of consistently and completely assessed alternatives. The example was deliberately constructed to show such a deficiency in the original AHP. In recent years the issue of rank reversal triggered many heated discussions in the research community (see Dyer (1990)). Lastly, we observe that SMART (the Simple Multi-Attribute Rating Technique, see Von Winterfeldt and Edwards (1986)) can be extended in a similar way as the Multiplicative AHP in order to deal with power relations in groups, although the method is additive and works with difference information, whereas the Multiplicative AHP uses ratio information.

Mathematically, there is no difference between the behaviour of the power coefficients and the criterion weights, at least in the proposed aggregation procedure. They are used as exponents in the weighted geometric means. This seems to make sense: aggregation over the criteria via geometric means is usually carried out in order to find a compromise between the conflicting preferential feelings within the mind of a single decision maker. Similarly, aggregation over the decision makers may be carried out in order to find a compromise between the preferential feelings within the group. The mathematical behaviour of the power coefficients therefore suggests that they are properly incorporated in the Multiplicative AHP via the weighted logarithmic least-squares method. In the original version of the present paper, this is illustrated by the power relations between member countries in the European Community, at least under the assumption that the Gross National Product, the size of the population, or the number of seats in the European Parliament is a proper yardstick for relative power.

2. Pairwise comparisons

In the basic experiment at the first evaluation level of the analysis, two stimuli S_j and S_k (two alternatives A_j and A_k under a particular criterion) are presented to decision maker d who is requested to express his graded comparative judgement, that is, to express his indifference between the two, or his weak, definite, strong, or very strong preference for one of them. We assume that the stimuli have unknown subjective values V_j and V_k which are the same for all decision makers in the group (otherwise, a compromise would hardly be possible whereas the group members are supposed to arrive jointly at a common group standpoint; see the bimodal symmetry mentioned in the introduction). The purpose of the basic experiments and the subsequent analysis is to approximate these values by the calculated impact scores. The verbal comparative judgement, given by decision maker d and converted into a numerical value r_{jkd}, is taken to be an estimate of the ratio V_j/V_k. Hence, since we only have ratio information, we may take the subjective values to be normalized in the sense that they sum to 1 or to 100%.

In the multiplicative variant we convert the gradations of the decision makers' comparative judgement into numerical values on a geometric scale which is conveniently characterized by the scale parameter γ. Thus, we set

$$r_{jkd} = \exp(\gamma\delta_{jkd}),$$

where δ_{jkd} is an integer-valued index designating the gradations of the decision maker's judgement as follows:

-8	very strong preference for S_k versus S_j,
-6	strong preference for S_k versus S_j,
-4	definite preference for S_k versus S_j,
-2	weak preference for S_k versus S_j,
0	indifference between S_j and S_k,
+2	weak preference for S_j versus S_k,
+4	definite preference for S_j versus S_k,
+6	strong preference for S_j versus S_k,
+8	very strong preference for S_j versus S_k.

Intermediate integer values may be assigned to the gradation index in order to express hesitations between two adjacent gradations. A plausible value of the scale parameter γ is given by ln 2, which implies that we are working on a geometric scale with progression factor 2 (see Lootsma (1993)).

Next, we approximate the vector V of subjective stimulus values via the logarithmic least-squares method. Introducing the set D_{jk} to denote the set of decision makers who judged S_j with respect to S_k we approximate the vector V by the normalized vector $\bar{v}$ which minimizes the function

$$\sum_{j<k} \sum_{d \in D_{jk}} (\ln r_{jkd} - \ln v_j + \ln v_k)^2 \, p_d, \tag{1}$$

where the power coefficient p_d stands for the relative power of decision maker d (the relative size of the constituency or the country represented by him). We take the power coefficients to be normalized so that they sum to 1. Introducing the quantities

$$q_{jkd} = \ln r_{jkd} = \gamma \delta_{jkd},$$

and

$$w_j = \ln v_j,$$

we can rewrite (1) as

$$\sum_{j<k} \sum_{d \in D_{jk}} (q_{jkd} - w_j + w_k)^2 p_d. \tag{2}$$

Using the properties

$$q_{jkd} = -q_{kjd} \text{ for any } j \text{ and } k,$$

$$D_{jj} \text{ is empty and } q_{jjd} = 0 \text{ for any } j,$$

we can write the associated set of normal equations in the form

$$R_j\, w_j - \sum_{k=1}^{n} C_{jk}\, w_k = \sum_{k=1}^{n} \sum_{d \in D_{jk}} q_{jkd}\, p_d \tag{3}$$

where the symbol

$$R_j = \sum_{k=1}^{n} \sum_{d \in D_{jk}} p_d$$

denotes the total power in row j of the matrix of pairwise comparisons, and the symbol

$$C_{jk} = \sum_{d \in D_{jk}} p_d$$

the total power in cell (j,k) of the matrix. A normalized solution minimizing (1) can now easily be found. We solve the linear equations (3) to obtain a solution w^* with an additive degree of freedom. Thereafter we calculate the vector v^* with components $v_j^* = \exp(w_j^*)$, and we use the multiplicative degree of freedom in v^* to find the normalized solution $\bar{v}$. In earlier papers, Lootsma (1987, 1993) has shown that the rank order of the components of $\bar{v}$ (and hence the rank order of the stimuli) does not depend on the scale parameter γ.

3. Aggregation by geometric means

When each decision maker expresses his opinion about every pair of stimuli, then

$$D_{jk} = \{1, \ldots\ldots, g\} \text{ for any } j \text{ and } k,$$

where g represents the size of the group. The normal equations (3) can now be reduced to

$$n \cdot w_j - \sum_{k=1}^{n} w_k = \sum_{k=1}^{n} \sum_{d=1}^{g} q_{jkd}\, p_d .$$

Since there is no unique solution to the system of normal equations we may set the sum of the variables to zero in order to obtain a particular solution. An unnormalized solution to the weighted logarithmic least-squares problem (1) can now explicitly be written as

$$v_j = \exp(w_j) = \sqrt[n]{\prod_{k=1}^{n} \prod_{d=1}^{g} r_{jkd}{}^{p_d}} . \tag{4}$$

Thus, we have a sequence of geometric means. When we average over row j in the pairwise comparison matrix of decision maker d, we obtain his estimate of the subjective value of stimulus S_j. When we average over the decision makers, we obtain the group preference for S_j with respect to S_k. Formula (4) yields the group's estimate of the subjective value of stimulus S_j.

We are now in a position to consider the role of the criteria. We start from the assumption that the (normalized) criterion weights c_i of the respective criteria C_i, $i = 1,...,m$, have been set in previous group discussions and that they are unanimously accepted by the members. Alternatively, these weights were already given at the time when the decision-making group was established (see also section 6). Let us now consider the basic experiment where decision maker d judges the alternatives A_j and A_k under criterion C_i, and let us take the symbol r_{ijkd} to represent the numerical value assigned to his verbal estimate of V_{ij}/V_{ik}, the ratio of the subjective values of the alternatives under consideration. The impact scores or partial scores a_{ij} of the alternatives A_j, $j=1,...,n$, under the respective criteria C_i, $i=1,...,m$, follow from geometric-mean calculations over the decision makers and over the rows of the pairwise-comparison matrices. Lastly, we use the geometric-mean aggregation rule of the Multiplicative AHP to obtain an unnormalized final score s_j for alternative A_j according to the expression

$$s_j = \prod_{i=1}^{m} a_{ij}^{c_i},$$

or, equivalently,

$$s_j = \sqrt[n]{\prod_{i=1}^{m} \prod_{k=1}^{n} \prod_{d=1}^{g} r_{ijkd}^{c_i p_d}}. \tag{5}$$

We have again a sequence of geometric means which can be computed in any order. Moreover, the intermediate results seem to make sense, whatever the order of the calculations is: averaging over the rows in the pairwise-comparison matrices of the respective decision makers and thereafter over the criteria, for instance, produces the final scores of the alternatives for each decision maker separately.

The original AHP does not have such a flexibility. True, it also "synthesizes" the pairwise comparisons of the individual decision makers via the calculation of geometric means per cell (Aczél and Saaty (1983)); next, however, it applies an eigenvector analysis to generate the impact scores of the alternatives under the respective criteria (the Multiplicative AHP uses geometric row means here); lastly, weighted arithmetic means (instead of geometric means) over the criteria yield the final scores of the alternatives. Alterations in the order of the calculations will generally not produce identical scores in the original AHP.

More recently, Saaty and Alexander (1989) proposed to use the original AHP with three hierarchical decision levels in order to identify the most likely outcome of open conflicts such as in Northern Ireland or in South-Africa before 1990, where the actors were not even in eye-to-eye contact. At the top level of the hierarchy one finds the parties involved in the conflict, at the intermediate level the objectives of the parties, and at the bottom level the outcomes in the form of alternative political structures. Via pairwise comparisons the parties are weighted according to their power to influence the outcome. Next, the objectives of each party are weighted according to their relative importance for the party. Finally, the alternative structures are compared in pairs according to how well each would satisfy a given objective in the view of the party. The authors use the arithmetic-mean aggregation rule of the original AHP on the simple ground that power is some sort of substance that can arbitrarily be fragmented and added together. Thus, one unit of power is divided among the parties, whereafter it flows downwards along the paths of the hierarchy. It is further divided among the objectives, and from the objectives it is again divided among the alternative structures. Addition of the fractions of power arriving at the respective alternatives yields the final scores indicating the probabilities that the structures will eventually emerge when the conflict ends. This model tacitly assumes that pairwise comparisons on the basis of relative power (parties, decision makers) and relative importance (objectives, criteria) are similar to those based on relative preference (alternatives). Section 5 will show that this is not correct. The model also suffers from the shortcomings which are due to the application of additive operations on quantities which represent ratio information.

Note that normalization of the impact scores and the final scores is a cosmetic operation which merely enhances the readibility of the calculated quantities. Since the decision makers are just requested to supply ratio information only (estimated ratios of subjective values under each of the criteria respectively), we cannot approximate the subjective values of the alternatives themselves but at most their ratios, under each of the criteria separately and under all criteria simultaneously. The fact that we use normalized criterion weights and normalized power coefficients does not imply that there would be even more degrees of freedom in the scores: we aggregate over the criteria and over the decision makers via weighted geometric means where normalized exponents are the rule. Thus, the calculated scores have a multiplicative degree of freedom only, which we can use to brush them up.

In the original version of this report we illustrate the three-level aggregation procedure by a generalized version (with three decision makers) of the example of Belton and Gear (1983).

4. Relationship with SMART

In an earlier paper the second author (1993) found a simple relationship between the Multiplicative AHP and the Simple Multi-Attribute Rating Technique (SMART, see Von Winterfeldt and Edwards (1986)). The basic idea is that the ratio estimates r_{ijkd} can also be obtained by direct rating on a numerical scale with equidistant echelons. The decision makers are merely requested, when they judge the alternatives under a particular criterion, to express their judgement by choosing an appropriate value between a preset lower anchor value for the worst (real or imaginary) alternative and a preset upper anchor value for the best (real or ideal) alternative. In schools and universities such a procedure is well-known as the assignment of grades expressing the performance of the pupils or students on a scale between 1 and 5, between 1 and 10, or between 1 and 100 (the upper anchor value varies from country to country; sometimes the scale is even upside down so that the grade 1 is used to express excellent performance). Concentrating on the scale between 1 and 10 we use the following values or grades to represent the performance of the alternatives under a given criterion:

10 excellent,

8 good,

6 fair,

4 poor,

2 extremely poor,

and we take the intermediate integer values to express judgemental transitions between these grades (9, very good, etc.). In pass-or-fail decisions at schools, the grades 4 and 5 are normally used for a poor performance that can still be compensated by high grades elsewhere. The grades 2 and 3 designate a really unacceptable pereformance. What matters in SMART and in the Multiplicative AHP, however, is the difference between grades. For the ratio estimates, we have

$$r_{ijkd} = \exp(\gamma(g_{ijd} - g_{ikd})),$$

where g_{ijd} and g_{ikd} stand for the grades assigned to the alternatives A_j and A_k under criterion C_i by decision maker d. We set $\gamma = \ln 2$, as we did in the Multiplicative AHP. According to formula (5) the final score of alternative A_j is now given by

$$s_j = \exp\left(\gamma \sum_{i=1}^{m} \sum_{d=1}^{g} c_i \, p_d \, g_{ijd} \right), \tag{6}$$

which shows that we only have to apply two arithmetic-mean calculations in an arbitrary order. The criterion weights and the power coefficients are the same, regardless of whether we use SMART or the Multiplicative AHP.

Unlike the AHP which elicitates detailed ratio information in a fragmented way, SMART collects difference information and it enables the decision makers to keep a holistic view on the set of alternatives. Working with SMART is usually faster. Moreover, because everybody has once been subject to his or her teacher's judgement, the grades are numbers with a strong qualitative connotation which can easily be used in multi-criteria analysis. Hence, SMART seems to be particularly useful in electronic meetings where the decision makers are under a heavy time pressure.

Obviously, the Multiplicative AHP and SMART are logarithmically related and SMART is purely additive. This enabled L. Rog (Delft University of Technology) to incorporate both methods in the REMBRANDT program for multi-criteria analysis. The user has the option to choose between the Multiplicative AHP and SMART, under each criterion again, so that ratio information is collected under some criteria (estimated ratios of subjective stimulus values) and difference information under the remaining ones (estimated differences of stimulus values expressed in orders of magnitude).

5. Scale values for relative power

Because power coefficients behave mathematically like the criterion weights, we can assign numerical scale values to verbal qualifications like *somewhat more, definitely more, much more*, and *vastly more* powerful in the same way as we quantified verbal statements like *somewhat more,......, vastly more* important (Lootsma (1993)). Consider a decision-making group with two members D_1 and D_2, and suppose that there are two real or imaginary alternatives A_j and A_k such that the decion makers have an equally strong but inverse preference for them under a particular criterion . These preferences are estimated by

$$\exp(\gamma\delta_{jk}) \text{ and } \exp(-\gamma\delta_{jk})$$

respectively, where δ_{jk} designates the selected gradation of D_1's comparative judgement. We assume that these preferences do not depend on the performance of the alternatives under the remaining criteria. Now, imagine that the group of the two decision makers jointly has a preference for A_j versus A_k estimated by

$$\exp(\gamma\theta_{jk}),$$

where θ_{jk} denotes the gradation which designates the group's preference. Taking ω to stand for the relative power (the ratio of the power coefficients) of D_1 with respect to D_2, we obtain by the geometric aggregation rule that

$$\exp\Big(\gamma \frac{\omega}{\omega+1} \delta_{jk}\Big). \exp\Big(-\gamma \frac{1}{\omega+1} \delta_{jk}\Big) = \exp(\gamma\theta_{jk}),$$

whence

$$\omega = \frac{\delta_{jk} + \theta_{jk}}{\delta_{jk} - \theta_{jk}}.$$

Note that ω does not depend on the scale parameter γ. It will be clear that $|\theta_{jk}| < |\delta_{jk}|$ because the group preference cannot absolutely exceed the preference expressed by the decision makers individually. It is easy to verify now that ω varies roughly between $\frac{1}{16}$ and 16 when δ_{jk} varies between -8 and 8, and θ_{jk} between the values $-|\delta_{jk}|$ and $|\delta_{jk}|$. The rather extreme value $\omega = 15$ is obtained when $\delta_{jk} = 8$ and $\theta_{jk} = 7$, which means that D_1's very strong preference for A_j with respect to A_k almost completely wipes out the equally strong but inverse preference of D_2. So, we take a ratio of 16:1 to stand roughly for a *vastly more* powerful position.

Assuming that geometric progression is also plausible for the gradations of relative power, we take the geometric sequence 1,2,4,8,16 to model *equal, somewhat more, definitely more, much more*, and *vastly more* power. If we allow for hesitations between two adjacent gradations of relative power we eventually have a geometric sequence of scale values with progression factor $\sqrt{2}$. The inverse echelons are, of course, taken to designate *equal, somewhat less,... , vastly less* power. These values could successfully be used in pairwise comparisons of the decision makers, when there are no simple, measurable, one-dimensional indicators for relative power, but only the verbal judgemental statements given by managers or administrators who are involved in the establishment of a decision-making committee. After a conversion of the statements into numerical values, power coefficients could be obtained in a similar way as criterion weights, and laid down in the committee's charter.

The above results show that one has to make a distinction between the relative power of decision makers and the relative importance of criteria on the one hand, and the relative preference for alternatives on the other. We represented the gradations of comparative judgement by a geometric sequence of scale values, but the progression factors are different, $\sqrt{2}$ for power and importance, and 2 for preference. The distinction emerges when we model relative power and relative importance via a particular experiment: the pairwise comparison of two alternatives by two decision makers jointly or under two criteria simultaneously. It is worth noting here that estimating the criterion weights and the power coefficients is politically more delicate than judging the alternatives. The choice of the criteria and the vague verbal formulation of their relative importance is usually felt to be the prerogative of those who established the decision-making committee (they could also compare the criteria in pairs in order to produce criterion weights). Relative power is even more sensitive so that it sometimes cannot be discussed, although it is observable throughout the decision process. We also note here that, just like the relative power of the decision makers, the relative importance of the criteria is a concept which exists in the mind of the decision makers, regardless of the physical or monetary units expressing the performance of the alternatives. The Multiplicative AHP and SMART properly model this concept via relative (not marginal!) substitution rates, to be discussed in a later paper.

A simple example to illustrate the possible significance of the ratio 16:1 assigned to *vastly more* power is given by the relative power positions (population size, gross national product, number of seats in the European Parliament) of the member countries in the European Community. This is shown in the original version of the present paper.

6. Concluding remarks

Let nobody have the illusion that the above incorporation of power relations would be welcome in applications of multi-criteria analysis! In a project of Lootsma et.al. (1990), carried out to the order of Directorate-General XII (Science, Research, and Development) of the European Commission, the proposal to weigh the judgemental statements of the participating decision makers by the population size or the GNP of their respective countries was immediately rejected. Such a multi-criteria analysis would have been possible already with an earlier version of the REMBRANDT program developed at the Delft University of Technology, but it was too early to model the on-going power game in the Community explicitly. In other projects, we have similar experiences. Although public administrators and industrial managers privately agree that it could be interesting to explore the relative power of various coalitions in a decision-making body, an open analysis of the power game still seems to be too hazardous in actual decision making. The situation is far from hopeless, however. Multi-criteria analysis has been and still is confronted with considerable resistance, which is partially due to the widespread feeling that a volatile concept like preference cannot properly be incorporated in a mathematical model. Similarly, an elusive concept like power would not be amenable to a mathematical analysis. Given the numerous applications of multi-criteria analysis, however, we fail to see why power relations should be unattainable for mathematical modelling.

Nevertheless, the road will not be easy, and there are particular stumbling blocks. In general, the literature on multi-criteria analysis ignores the question for which type of decision makers the respective methods have been designed. For the charismatic leader? That is difficult to believe. For the cool, balanced administrator? That is possible. For the manipulating gamesman (Maccoby (1976))? That is unlikely, but he may occasionally turn to the formalized approach of multi-criteria analysis. The literature also ignores the cultural background of the decision makers, although there are several dimensions to categorize and to understand their behaviour, such as individualism versus collectivism, large versus small power distance, strong versus weak uncertainty avoidance, and masculinity versus femininity (Hofstede (1984)). Finally, the recently developed information technology for group decision support may have a significant and unexpected impact. Extrovert decision makers tend to drop out of electronic meetings because they are dismayed with their inability to use their strong verbal skills in the voting and calculation mechanisms (LaPlante (1993)). What it all means is that, without a taxonomy of the decision makers and their typical behaviour in a decision process, it may be difficult to distinguish the potentially successful applications of multi-criteria analysis from the hazardous ones.

In fact, although we considered three decision levels, we do not really propose a hierarchical model. We only formalized the concept and the gradations of relative importance and relative power via a model which is based upon the pairwise comparison of alternatives under some (two or three) criteria and by some (two or three) decision makers simultaneously. This implies, legitimately, that the criteria and the decision makers are dependent on the alternatives. Criteria in a given decision problem are selected top down, on the basis of previous experience in related problems, and bottom up, on the basis of particular properties of the alternatives under consideration. Similarly, members of a decision-making committee are selected, not only on the basis of their position in the public or private organisation, but also on the basis of their ability to judge at least some of the alternatives under the prevailing criteria. So, criteria are not subordinate to decision makers, and vice versa. There is no simple top down hierarchy in a decision process: it may go through a number of cycles where alternatives, criteria, and decision makers are introduced and/or dropped, until an acceptable compromise solution has been reached.

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It is a pleasure to acknowledge Leo Rog (Delft University of Technology) for the development of the versatile REMBRANDT program for multi-criteria decision analysis. It does not only work with the Multiplicative AHP but also with the logarithmically related method SMART. The relationship is indicated by the acronym which stands for Ratio Estimation in Magnitudes or deci-Bells to Rate Alternatives which are Non-DominaTed. We have successfully used the optional weighing of the decision makers in order to check the hand calculations in the example of Belton and Gear and to vary the power coefficients.

References

[1] Aczél, J., and Saaty, Th.L., "Procedures for synthesizing ratio judgements". *Journal of Mathematical Psychology* 27, 93-102, 1983.

[2] Barzilai, J., "On the use of the eigenvector in the AHP". *Proceedings of the X-th Int. Conf. on MCDM,* Taipei, Taiwan, Volume I, pp. 291-300, 1992.

[3] Barzilai, J., Cook, W.D., and Golani, B., "Consistent weights for judgement matrices of the relative importance of alternatives". *Operations Research Letters* 6, 131-134, 1987.

[4] Barzilai, J., and Golani, B., "An axiomatic framework for aggregating weights and weight-ratio matrices". *Proceedings of the Second Int. Symp. on the AHP,* Pittsburgh, Penn., pp. 59-70, 1992.

[5] Belton, V., and Gear, A.E., "On a shortcoming of Saaty's method of analytical hierarchies". *Omega* 11, 227-230, 1983.

[6] Dyer, J.S., "Remarks on the Analytic Hierarchy Process". *Management Science* 36, 249-258, 1990. In the same issue there are apologies by Th.L.Saaty (259-268), P.T.Harker and L.G.Vargas (269-273), and a further clarification by J.S.Dyer (274-275).

[7] Galbraith, J.K., "*The Anatomy of Power*". Houghton Mifflin, Boston, 1983.

[8] Jacobs, F., Corbett, R., and Shackleton, M., "*The European Parliament*". The Longman Group, Harlow, Essex, UK, 2nd edition, 1992.

[9] Hofstede, G.H., "*Culture's consequences, international differences in work-related values*". Sage, Beverly Hills, 1984.

[10] LaPlante, A., "Nineties Style Brainstorming". *Technology Supplement to Forbes Magazine,* October 25, pp. 44-61, 1993.

[11] Lootsma, F.A., "Numerical scaling of human judgement in pairwise-comparison methods for fuzzy multi-criteria decision analysis". In G.Mitra (ed.), *Mathematical Models for Decision Support.* Springer, Berlin, pp. 57-88, 1988.

[12] Lootsma, F.A., "Scale sensitivity in the Multiplicative AHP and SMART". *Journal of Multi-Criteria Decision Analysis* 2, 87-110, 1993.

[13] Lootsma, F.A., Mensch, T.C.A., and Vos, F.A., "Multi-criteria analysis and budget reallocation in long-term research planning". *European Journal of Operational Research* 47, 293-305, 1990.

[14] Maccoby, M., "*The gamesman, the new corporate leader*". Simon and Schuster, New York, 1976.

[15] Saaty, Th.L., "*The Analytic Hierarchy Process, planning, priority setting, and resource allocation*". McGraw-Hill, New York, 1980.

[16] Saaty, Th.L., and Kearns, K.P., "*Analytical planning, the organization of systems*". Pergamon, New York, 1985.

[17] Saaty, Th.L., and Alexander, J.M., "*Conflict resolution, the Analytic Hierarchy approach*". Praeger, New York, 1989.

[18] Winterfeldt, D. von, and Edwards, W., "*Decision analysis and behavioral research*". Cambridge University Press, Cambridge, UK, 1986.