Integrating AHP and Traditional OR/MS Methodologies

Ernest H. Forman
School of Government and Business Administration
George Washington University, Washington D.C. 20052

ABSTRACT

This paper illustrates how the Analytic Hierarchy Process (AHP) can be integrated with traditional OR/MS methodologies to provide a decision support system that is understandable and relevant to real world decision makers. It shows how decision makers can develop, understand and utilize models for decision making, something they rarely do today. Examples showing the integration of AHP with linear programing, queueing analysis, critical path method, forecasting techniques, and integer linear programming are discussed.

I) INTRODUCTION

Numerous textbooks are devoted to Operations Research / Management Science (OR/MS) methods and applications (Anderson, 1986). Although Operations Research and Management Science have provided substantial benefits to corporations and governments over the past half century, many roadblocks have prevented the realization of their ultimate potential in the decision making process. This paper will illustrate how the Analytic Hierarchy Process (AHP) (Saaty 1980, 1982) can be integrated with traditional OR/MS methodologies to provide decision support that is understandable and relevant to real world decision makers.

We will illustrate with examples integrating the use of AHP with linear programming, queueing, critical path method, forecasting, and integer linear programming

I) INTEGRATION OF LINEAR PROGRAMMING AND AHP -- PRODUCT DESIGN

The idea for a new product must be developed into specifics. There are usually numerous alternatives for designing each "piece" of a product, and the problem of choosing the "best" design from a very large combination of alternatives can be overwhelming. Traditional textbook examples illustrate how linear programming can be helpful in selecting the best combination of components for a product. Consider a problem of selecting plastic body materials for a new Sporty Convertible being designed by an auto manufacturer.

A traditional linear programming formulation might consist of an objective function to minimize costs, subject to constraints on

1) body weight -- that the body weight be no more than 120 pounds,
2) coverage -- that there be at least 5 cubic feet of body material in order to cover the body, and
3) strength -- that the mixture of materials possess a strength of at least 100 pounds per square foot.

Let's reflect on where some of the relationships and parameters for this model would come from in a real world application.

Minimizing cost is obviously an objective, and the coefficients of the objective function, representing the cost per cubic foot of material can be obtained from a data base of suppliers. The fact that we "must" have at least 5 cubic feet of body material to cover the body can come from preliminary design drawings of the sport convertible.

But the constraint requiring that the body weigh no more than 120 pounds is somewhat contrived since one might argue that a light body weight is an "objective" (rather than a constraint) and that we really do not know what we "must" have as a maximum body weight. We would like the body to weigh as little as possible so that the car will accelerate better and use less fuel. Then why was body weight represented as a constraint in the traditional formulation? Simply as a convenience, because linear programming allows only one objective, and we had already chosen cost minimization as that objective.

Similarly, the desire to have as strong a body material as possible is an objective, not a constraint. Almost every real world decision involves multiple objectives. Many of the constraints in LP problem formulations are actually objectives in disguise[1] and are included because LP formulations are limited to one objective. In the Sporty Convertible example, we had already chosen cost minimization as that objective. Thus we attempted to achieve a weight objective by including a constraint that the body material must weigh no more than 120 pounds. However, this approach is not really adequate. Specifying a value of 120 is somewhat arbitrary. Why not 100, or 150?

Many of the constraints in LP problem formulations are actually objectives in disguise. Therefore, a pure linear programming approach to this problem, which allows *only one objective*, appears to be "forced" and there is strong likelihood that senior management will, rightfully, feel uncomfortable with the analysis and not make proper use of it in their decision.[2 3]

Let's see how a decision maker might actually approach this decision. It would be highly unlikely that he would begin with a linear programming formulation in mind. Instead, he might query a data base of body materials and be presented with the following information:

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[1]Some are "both", a constraint on some minimal (or maximal) value, and an objective to achieve as much (or as little) beyond that.

[2]We believe that this is the major reason that LP has not been used more extensively in practice.

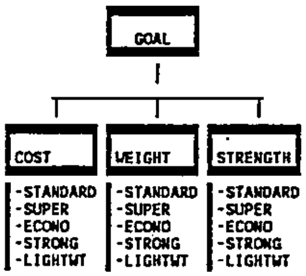
[3]Another approach to overcoming the limitation of only one objective in an LP solution is called Goal programming. See (Dyer, 1988) for a discussion of the limitations of this approach.

Material	1	2	3	4	5
Description	Standard	Super	Econo	Strong	Lightwt
Cost/Cu Ft	105	220	85	103	107
Weight/Cu Ft.	25	15	40	55	15
Strength Lbs/Sq-Ft	20	35	11	42	12

He might then begin to list the pros and cons of each material. For example, the "pro" for the "Econo" material is its low cost per cubic foot. However, it has two "cons": it is relatively heavy, and it is not very strong. The "Lightwt" material's pro is its relatively low weight, but its cons are its moderately high cost (at least higher than the "Econo" material) and its relatively low strength. The "Super" material has two pro's: its relatively low weight and relatively high strength. It also has one important con: its very high cost.

The decision maker might then ponder how to evaluate the tradeoffs between the pros and cons. If he were astute, he might develop a methodology similar to one developed by Benjamin Franklin over two hundred years ago[4]. The Analytic Hierarchy Process (AHP) is a more recent and improved approach. From the pros and cons, the decision maker can identify three primary objectives: (low) cost, (low) weight, and (high) strength. An AHP model with these objectives and the five alternatives is shown below.

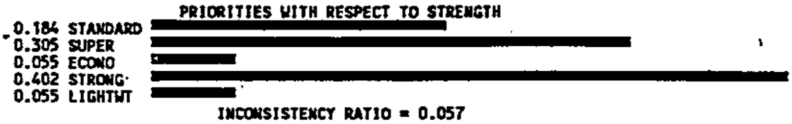
Design "Best" Sporty Convertible Body Material



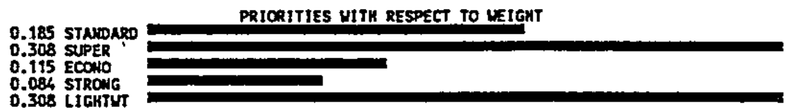
The relative preferences of the alternatives can be determined by the decision maker using not only the hard data about the materials, but quite probably subjective judgments about the utility of the characteristics represented by the hard data as well. For example, when making judgments with respect to the strength criterion, the decision maker might refer to the figures above and, using his previous experience, judge that the "Econo" and "Lightwt" materials are about EQUALLY preferable in spite of the fact that the "Lightwt" is just a little bit stronger; he might then judge that the "Standard" material is STRONGLY more preferable to either, and the "Strong" material is only MODERATELY more preferable to the "Standard" material.

These judgments and the judgments for other pairs of alternatives with respect to Strength resulted in the following priorities:

[4]Letter from Benjamin Franklin to Joseph Priestly in 1722, Benjamin Franklin Sampler, 1956

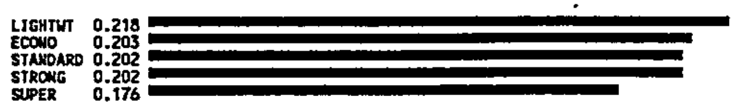


Preferences with respect to Cost and Weight were developed using Expert Choice (Forman 1983) "Compare-Other-What-if" command and specifying numerical values equal to the reciprocals of the costs and weights of the materials. The resulting priorities were:



Next, pairwise comparisons for the relative importance of the three criteria were made with the resulting priorities:

Finally, a synthesis of the priorities of the five materials over the three criteria resulted in the following:



These priorities represent the overall relative "effectiveness" of the alternatives with respect to the three criteria. It is interesting to observe how close these measures of "effectiveness" are, especially for the top four alternatives, particularly since there were such significant differences with respect to the individual criteria.

Based on these measures of "effectiveness", we can formulate an LP model to determine the composition, at first using only the constraint that we must have five cubic feet of body material to cover the frame. Since the number of basic variables in an LP is equal to the number of constraints, it is not surprising that the "optimal" solution is to use five cubic feet of the plastic with the highest measure of effectiveness, the lightweight plastic.

	STANDARD	SUPER	ECONO	STRONG	LIGHTWEIGHT[5]
EFFECTIVENESS	0.202	0.176	0.203	0.202	0.218
DECISION VARIABLES	0.00	0.00	0.00	0.00	5.00

OVERALL EFFECTIVENESS	1.09
COST	535.00
WEIGHT	75.00
STRENGTH	60.00

Now we must examine the solution. Is there anything that appears missing or wrong? If so, we must include additional objectives or constraints. For example, in the above, we have treated low weight and high strength more naturally as objectives, rather than as constraints as in the traditional LP formulation. But if we implement this solution, what will the cost, weight and strength characteristics of the body material be?

The cost is $535, the body weight is 75 pounds, and the strength is 60 pounds/per square foot. The latter is <u>well</u> below the 100 pounds originally thought to be the minimum required. So let's now add a constraint specifying that the minimum strength should be 100 pounds. The modified LP results in an optimum solution of 1.33 cubic feet of the strong and 3.67 cubic feet of the lightweight.

	STANDARD	SUPER	ECONO	STRONG	LIGHTWEIGHT
EFFECTIVENESS	0.202	0.176	0.203	0.202	0.218
DECISION VARIABLES	0.00	0.00	0.00	1.33	3.67

OVERALL EFFECTIVENESS	1.07
COST	529.67
WEIGHT	128.33
STRENGTH	100.00

Not only has the strength increased to the required 100 pounds, but the cost has actually decreased from $535 to $529.57. This has been achieved by increasing the body weight from 75 pounds to 128.33 pounds. Since 128.33 body weight is acceptable, this solution is accepted as "optimal".

In comparing this multicriteria solution to that obtained with the traditional single criterion LP approach -- of minimizing cost subject to (somewhat arbitrary) constraints, it is seen that the multicriteria approach allows us to trade off cost versus weight, since this solution is lower in cost ($529.57 vs. $545.65) but heavier (128.33 lbs. vs. 120 lbs.) In addition, this solution uses a mixture of only two plastics as opposed to three for the traditional solution, a simplification that might result in additional savings as well.

In summary, the multicriteria approach consists of using AHP to derive measures of effectiveness for the alternatives considering more than just the single objective, cost. It then uses LP with only the obvious constraint(s) (in this case body coverage). The tentative

[5]The linear programming models were solved in a Lotus 1-2-3 spreadsheet format using the What'sBest software (Savage 1985). What'sBest is an excellent way to (re)introduce decision makers to linear programming or integer linear programming because it places the optimization process in the context of a spreadsheet formulated according to the decision makers view of the world.

solution is then examined to see if it is reasonable. If not, because one or more "must" objectives are obviously not met (in this example an insufficient body strength), new constraint(s) are introduced for the emerging "must(s)" and the LP solved again. In addition, judgments in the AHP model that are used to derive the measures of effectiveness of the alternatives can be re-evaluated in light of the knowledge gained by looking at the tentative solution. Iteration continues until an "optimal" solution satisfying the multiple objectives is achieved.

II) AHP AND QUEUEING FOR RESOURCE ALLOCATION DECISIONS

Resource allocation decisions attempt to "balance" the benefits derived from the allocation of resources with the costs of those resources. Although many attempts have been made to quantify all benefits and costs, it is hardly ever possible to do so. We will look at two resource allocation examples, one involving a queueing analysis and the other a critical path calculation.

Consider first the problem of deciding how many draftsmen with CAD/CAM equipment to provide for the design of new products[6]. A queueing analysis provides the following measures of performance based on the number of servers (draftsmen with CAD/CAM equipment):

Number of Servers	Probability a request will have to wait	Average # of requests in queue	Average time to complete service
1	.95	18.05	20.00 hours
2	.31	0.28	1.29 hours
3	.01	0.04	1.03 hours

Having determined this, the question remains, how many servers should be used? The queueing analysis only helped to derive some measures of effectiveness. It did not really answer the question. One way to begin to answer this question and arrive at a decision is to list the pros and cons of each of the three alternatives as follows:

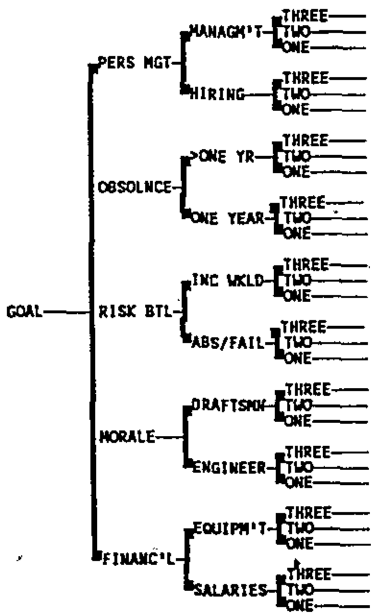
[6]This example is taken from (Dyer 1989).

Altern-ative	PROS	CONS
One Server	Low Expense for draftsmen Low expense for CAD/CAM equipment	Draftsmen will see many requests waiting in line and become dejected, feeling that he will never get caught up Engineers will become frustrated because they almost always have to wait for a draftsman to start their job Engineers will become frustrated because the average time to get their job back from the draftsmen will be 20 hours, or 2 1/2 days! Risk of machine failure or absent draftsman -- what happens--if one CAD/CAM machine goes down and or a draftsman does not come in to work?
Two Servers	Draftsmen will see a great reduction in requests waiting in line Engineers will see a great reduction in the wait for the completion of their job --from 20 hours to 1.3 Engineers will have wait for a draftsman to start on their job only about 31% of the time Decreased risk due to machine failure or absent draftsmen-- if one CAD/CAM machine goes down or a draftsman does not come to work there will still be a backup Increased expense for draftsmen Increased expense for CAD/CAM equipment Increased exposure to obsolescence. If better machines become available, it is better to have as few on hand as possible so they can be scraped and replaced with new ones Exposure to risk of sharp degradation of service if the workload (arrival rate) increases	Increased expense for draftsmen Increased expense for CAD/CAM equipment Increased exposure to obsolescence. If better machines become available, it is better to have as few on hand as possible so they can be scraped and replaced with new ones Exposure to risk of sharp degradation of service if the workload (arrival rate) increases
Three Servers	Draftsmen will see a further reduction in requests waiting in line Engineers will have to wait for a draftsman to start on their job only about 1% of the time. Decreased risk due to machine failure or absent draftsmen-- if one or two CAD/CAM machines goes down or draftsmen do not come in to work there will still be backup(s) Engineers will see some reduction in the wait for the completion of their job --from 1.3 to 1 hour No sharp degradation of service even if the workload (arrival rate) increases	Increased expense for draftsmen Increased expense for CAD/CAM equipment Increased exposure to obsolescence. If better machines become available, it is better to have as few on hand as possible so can scrap them to keep up with technology Hiring difficulties for skilled draftsmen that know how to use the CAD/CAM system Managing three draftsmen will require more coordination and effort

Let us see how AHP can help management make the decision on how many draftsmen with CAD/CAM systems to use. The criteria for the decision can be extracted directly from the pros and cons:

- Financial considerations
  - Salaries of draftsmen
  - Expense of CAD/CAM equipment
- Morale (and stifling of creativity)
  - Engineers
  - Draftsmen
- Risk of bottlenecks and degradation of service due to absence of draftsmen or failure of CAD/CAM equipment
  - Increased workload
- Exposure to obsolescence of CAD/CAM equipment
  - Within one year
  - More than one year
- Personnel management
  - Hiring draftsmen
  - Managing draftsmen

A rational decision about how many servers (draftsmen with CAD/CAM equipment) to use must be based on criteria such as these. The decision will follow from an AHP model and judgments about the relative preferences of the alternatives with respect to these criteria and about the relative importance of the criteria. These judgments will be based partly on the results of the queueing analysis (as elaborated in the pros and cons) and partly on the knowledge and experience of the decision maker.

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>ONE YR  --- OBSOLESCENCE AFTER 1 YEAR
ABS/FAIL --- ABSENCE OF DRAFTSMEN OR FAILURE OF EQUIPMENT
DRAFTSMN --- DRAFTSMEN MORALE AND EFFICIENCY
ENGINEER --- ENGINEER MORALE, CREATITIVITY AND EFFICIENCY
EQUIPM'T --- COST OF CAD/CAM EQUIPMENT
FINANC'L --- FINANCIAL CONSIDERATIONS (DRAFTSMEN AND MACHINES)
HIRING   --- DIFFICULTY IN HIRING DRAFTSMEN WITH CAD/CAM EXPERTISE
INC WKLD --- RISK OF BOTTLENECKS DUE TO INCREASE IN WORKLOAD
MANAGM'T --- DIFFICULTY IN MANAGING TWO OR THREE DRAFTSMEN
MORALE   --- MORALE AND EFFECTIVENESS OF ENGINEERS AND DRAFTSMEN
OBSOLNCE --- EXPOSURE TO HAVING INVESTMENT IN OBSOLETE EQUIPMENT
ONE      --- ONE DRAFTSMAN - CAD/CAM SYSTEM
ONE YEAR --- OBSOLESCENCE WITHIN ONE YEAR
PERS MGT --- PERSONNEL MANAGEMENT (OF DRAFTSMEN)
RISK BTL --- RISK OF BOTTLENECKS DEVELOPING DUE TO ABSENCE/FAILURE
SALARIES --- SALARIES OF DRAFTSMEN
THREE    --- THREE DRAFTSMEN - CAD/CAM SYSTEMS
TWO      --- TWO DRAFTSMEN - CAD/CAM SYSTEMS
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III) AHP AND CPM FOR RESOURCE ALLOCATION DECISIONS

Consider the decision that management faces in deciding what resources to apply to a project in order to complete the project in as short a time as is "practically possible". Part of what management will consider practically possible will involve tradeoffs between time, money, labor, and materials.

The Critical Path Method (CPM) is a well known Operations Research technique used in project management. CPM is useful in analyzing the precedence relationships in a project of many activities, in determining which activities are on the "critical path", and determining how long it will take to complete the project. In addition, CPM can indicate where to apply additional resources if management desires to "crash" activities (speed them up) in order to reduce the total project completion time. CPM provides a great deal of useful information such as the following time, cost tradeoffs. This information makes it clear that a decision must be made -- management must choose the level of crashing from the available alternatives.

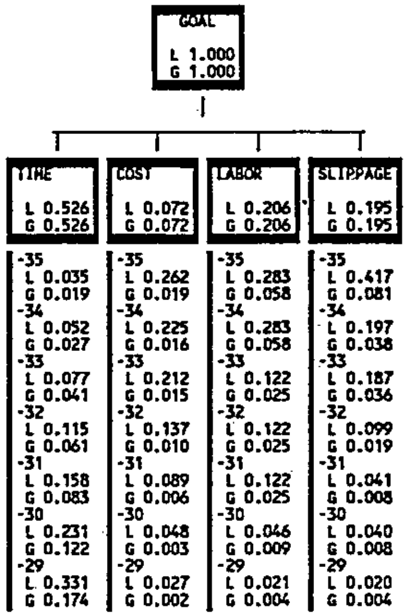
	Months to Complete	Project Cost ($000's)
Alternative 1	35	16,814.00
Alternative 2	34	16,820.67
Alternative 3	33	16,827.33
Alternative 4	32	16,845.33
Alternative 5	31	16,873.00
Alternative 6	30	16,942.00
Alternative 7	29	17,027.00

At first, the choice in this example appears relatively easy. The difference in cost between a project completion period of 35 months and 29 months is slightly more than two hundred thousand dollars out of a total of about 17 million dollars. Expressed as a percentage, this is not a large amount. But relative to managements discretionary budget it may be very large. Thus, considering only the cost criterion, it is not clear what management would decide. In addition, there are other factors that must be considered when trying to speed up a project, such as increases in labor stress and the probability of slippage. Thus, management should consider criteria such as the following in deciding which of the alternatives to choose:

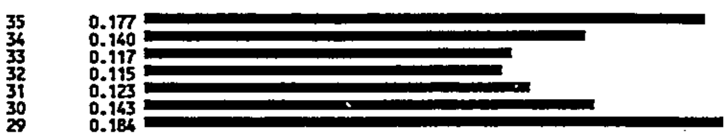
Months to Complete	Project Cost ($000's)	Labor Stress	Likelihood of Slippage
35	16,814.00	Low	Very little
34	16,820.67	Low	Low
33	16,827.33	Moderate	Low
32	16,845.33	Moderate	Moderate
31	16,873.00	Moderate	High
30	16,942.00	High	High
29	17,027.00	V. High	V. High

This decision, as almost all decisions, depends on both quantitative considerations (months to complete and project cost) and qualitative considerations (labor stress, likelihood of slippage, and the relative importance of the four criteria). While the CPM analysis has helped in determining the numerical tradeoffs between time to completion and project cost, it is only a <u>part</u> of the decision support <u>system</u>. An AHP analysis using the time, cost, labor stress and the likelihood of slippage can make use of the CPM analysis results in supporting management's decision making.

DETERMINE BEST LEVEL OF CRASHING



29 --- 29 MONTHS AT 17,027,000, V. HIGH STRESS, V. HIGH CHANCE SLIPPAGE
30 --- 30 MONTHS AT 16,942,000, HIGH STRESS, HIGH CHANCE OF SLIPPAGE
31 --- 31 MONTHS AT 16,873,000, MODERATE STRESS, HIGH CHANCE OF SLIPPAGE
32 --- 32 MONTHS, $16,845,330, MODERATE STRESS, MODERATE CHANCE SLIPPAGE
33 --- 33 MONTHS AT $16,827,330, MODERATE STRESS, LOW CHANCE OF SLIPPAGE
34 --- 34 MONTHS AT $16,820,670, LOW STRESS, LOW CHANCE OF SLIPPAGE
35 --- 35 MONTHS AT $16,814,000, LOW STRESS, V. LITTLE CHANCE OF SLIPPAGE
COST --- COST TO COMPLETE PROJECT
LABOR --- STRESS ON PERSONNEL
SLIPPAGE --- LIKELIHOOD OF SLIPPAGE
TIME --- TIME TO COMPLETE PROJECT



IV) AHP AND FORECASTING

Let us now look at how AHP can be useful in synthesizing information in order to make better decisions under conditions of uncertainty. We will illustrate two basic ideas. Although uncertainty cannot be eliminated, we will show how AHP can be used to derive probability distributions which, in essence, remove the uncertainty about uncertainty. Then we will show how AHP can be used to combine forecasts (in the form of probability distributions) from a variety of factors and/or techniques.

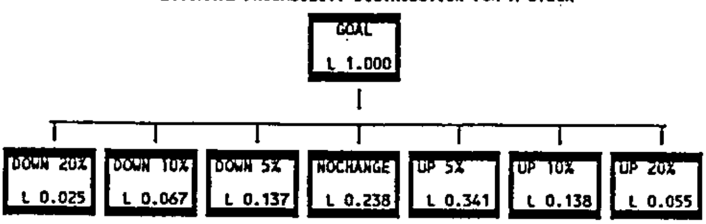
Consider an investor who is evaluating alternative stocks or options. The investor, after doing research, will form an opinion that a particular stock is likely to go up, or down. Suppose an investor is considering two alternative stocks and thinks each will go up. Is one more likely to go up than another? It may be that the investor feels that stock A is more likely to go up than stock B but that stock B has a greater probability of going up more than 20% than does stock A. How can the investor incorporate these feelings into his decision process? If the investor could translate his knowledge about the stocks into probability distributions, he could then use the probability distributions in choosing among the stocks, or in even more complex decisions, choosing among alternative strategies for stock option puts and calls.

It would be unreasonable to expect the investor to directly specify the probability distribution for a stock's price performance (over a specified period of time). However, it is rather natural for the investor to express his feelings about the anticipated stock's price performance via pairwise relative comparisons. For example, the investor should be able to translate his research about a stock into a judgment such as:

> the likelihood of a stock going up 5% in a given period of time is moderately more likely than the stock remaining at the current price, and
>
> the likelihood of a stock remaining unchanged is moderately to strongly more likely than going up 20%.

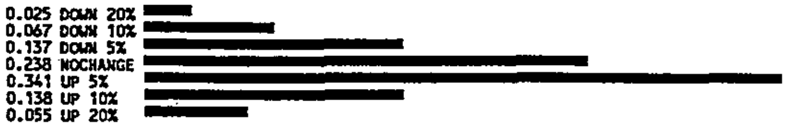
Just as redundancy (in the pairwise comparisons) has been shown to produce accurate estimates of quantifiable physical phenomena (such as area or intensity of light), the redundancy in the investors set of pairwise comparisons will result in probabilities that reflect the investors judgments, which in turn are based on his research as well as experience. In making the pairwise comparisons, the investor will find himself pressed to "think hard" and forced to question both his assumptions as well as the validity of his data. A typical set of comparisons along with the resulting probability distribution is shown below:

ESTIMATE PROBABILITY DISTRIBUTION FOR A STOCK



JUDGMENTS AND PRIORITIES WITH RESPECT TO
GOAL TO ESTIMATE PROBABILITY DISTRIBUTION FOR A STOCK

	DOWN 20%	DOWN 10%	DOWN 5%	NOCHANGE	UP 5%	UP 10%	UP 20%
DOWN 20%		(7.0)	(8.0)	(9.0)	(5.0)	(5.0)	(3.0)
DOWN 10%			(4.0)	(5.0)	(4.0)	(3.0)	1.0
DOWN 5%				(3.0)	(3.0)	1.0	2.0
NOCHANGE					(3.0)	2.0	4.0
UP 5%						3.0	6.0
UP 10%							4.0
UP 20%							



The translation of the investors research into a subjective probability distribution is a significant accomplishment since this probability distribution can subsequently be used to evaluate investment alternatives (using criteria meaningful to the investor, such as expected value, standard deviation, and the probability of gaining or losing more than a specified percent, along with other factors about the company such as its quality of management). In a sense, it can be said that this process of deriving a probability distribution "removes the uncertainty about uncertainty" by translating fuzzy feelings (e.g. the research indicates it will probably go up a little, or a lot) into a distribution of probabilities.

The above approach can be easily expanded to accommodate judgments based on specific factors and to synthesize forecasts derived from different forecasting perspectives. Four common perspectives to forecasting stocks/options/futures are:

Fundamental analysis (companies fundamentals, price earnings rations, supply, demand, etc.)

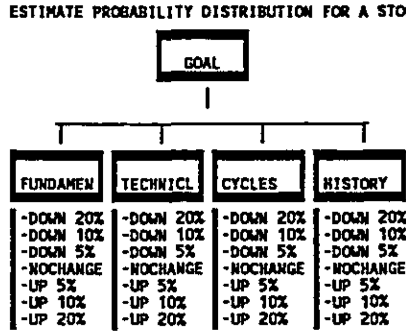
Technical analysis (charts, moving averages, support and resistance levels, Elliot waves, etc.)

Cyclical analysis

Historical analysis (what is the price relative to its historical highs, lows, etc.)

Some professional analysts use only one perspective, while others use a combination, trying to synthesize in their heads the likelihoods indicated by each perspective, and the relative importance they attach to each perspective at a particular point in time. This can be done with AHP as follows:

ESTIMATE PROBABILITY DISTRIBUTION FOR A STOCK

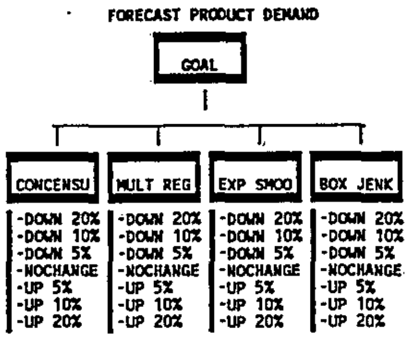


CYCLES --- CYCLICAL ANALYSIS
FUNDAMEN --- FUNDAMENTAL ANALYSIS (SUPPLY / DEMAND, COMPANY FUNDAMENTALS)
HISTORY --- HISTORICAL PERSPECTIVE
NOCHANGE --- NOCHANGE
TECHNICL --- TECHNICAL ANALYSIS, CHARTS, SUPPORT, RESISTANCE, ELLIOT WAVES

As another example, in forecasting future demand for a product, alternative approaches might consist of (Dyer 1986):

o consensus asking a group of experts to come to a consensus on judgments about relative likelihoods (perhaps by using a Delphi approach),
o multiple regression,
o exponential smoothing,
o Box Jenkins time series analysis

The synthesis of these techniques can be accomplished with AHP. Recent research has indicated that a combination of forecasting approaches produces better results than using only one approach (Conroy 1987).



BOX JENK --- BOX JENKINS
CONCENSU --- CONSENSUS OF A GROUP OF EXPERTS
EXP SMOO --- EXPONENTIAL SMOOTHING
MULT REG --- MULTIPLE REGRESSION

Given conditions present at any point in time, judgments about the relative importance to be given to the respective approaches in the above two examples can be made in a gestalt fashion. Alternatively, meaningful criteria such as recent success, long term success, success in the current economic environment can be used to determine how much reliance to place on each of the forecasts.

V) AHP AND INTEGER LINEAR PROGRAMMING FOR RESOURCE ALLOCATION

Suppose that we must decide on the appropriate combination of products to produce, subject to certain restrictions, such as budget limitations, diversification constraints, and dependency constraints. If we try to investigate each possible combination of products, two difficulties arise. First, how do we estimate the "overall" worth to the firm of a product or a specific combination of products? And second, if there is a relatively large number of products, the number of combinations is extremely large. For example, if we had 20 products and 10 constraints, we would have to consider more than 30 million combinations![7]

[7]Examining only the extreme points of the convex hull would require $(m+n)!/(m! \times n!)$ or $30!/(20! \times 10!)$points to be examined.)

Both these difficulties can be overcome using sophisticated DSS tools. The second difficulty is eliminated by formulating an Integer Linear Programming (ILP) mathematical model. ILP is similar to linear programming except that some of the variables must be integer, or specific integers, such as 0 and 1.[a]

Our problem can be formulated by defining decision variables Xi, i=1 to n, corresponding to the n products under consideration, where Xi will be equal to one if the ith product is to be produced, and zero if it is not. If we had a measure of the overall "worth" of each product to the firm, say Wi for the ith product, then we would like to maximize the sum of the worth over all products that will be included in the company's portfolio. This can be expressed as:

Maximize W1X1+W2X2+....WnXn (the worth of the products to be produced)

Subject to:

Budgetary constraint:

Diversification constraints:
(i.e. at least one product in each market segment, and
no more than two products in each market segment

Dependency constraints:
(e.g. either both products 1 and 2 or neither)

and Xi= 0 or 1.

The remaining difficulty, that of evaluating the worth (W_i) of each of the products can be solved using AHP. This approach allows one to consider all relevant considerations in the process of determining the "best" combination of products to produce.

Similar decisions to choosing a portfolio of products are the decision of which R&D projects to fund, and the decision of which magazines should be used for a marketing campaign.

Let us consider the choice of magazines for an advertising campaign for a 35 mm camera. Using an AHP model with the ratings approach, we can develop measures of effectiveness for each magazine with respect to objective criteria, such as income and age demographics of the readers of the magazines, as well as subjective criteria, such as editorial content. The figures below illustrate an Expert Choice model used to derive such measures of effectiveness.

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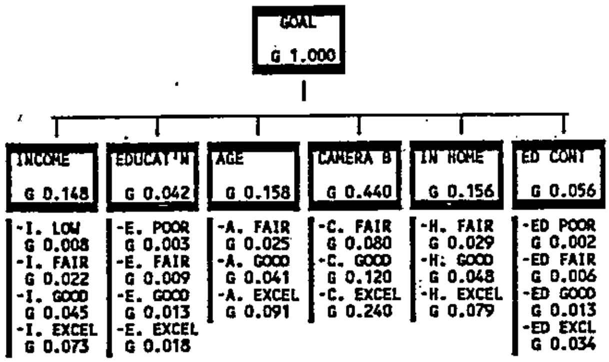
[a]Intuitively, it might appear that the problem is easier if some of the variables are constrained to be integer rather than allowed to take on any of a continuous range of values, but just the opposite is true.)

PRIORITIZE CRITERIA TO RATE MAGAZINES



A. EXCEL --- MAGAZINE WITH > 60% READERS IN 18-44 AGE BRACKET IS EXCELLENT
A. FAIR --- MAGAZINE WITH 40-50% READERS IN 18-44 AGE BRACKET IS FAIR
A. GOOD --- MAGAZINE WITH 50-60% READERS IN 18-44 AGE BRACKET IS GOOD
AGE --- % READERSHIP BETWEEN THE AGES OF 18 AND 44 YEARS OLD
C. EXCEL --- MAGAZINE WITH >20% READERS PURCHASING CAMERA <1 YR IS EXCELLENT
C. FAIR --- MAGAZINE WITH 10%-15% READERS PURCHASING CAMERA <1 YR AGO IS FAIR
C. GOOD --- MAGAZINE WITH 15-20% READERS PURCHASING CAMERA <1 YR IS GOOD
CAMERA B --- % READERSHIP THAT WERE CAMERA BUYERS WITHIN LAST 12 MONTHS
E. EXCEL --- MAGAZINE WITH >60% READERS HAVING ANY COLLEGE IS EXCELLENT
E. FAIR --- MAGAZINE WITH 40-50% OF READERS HAVING ANY COLLEGE IS FAIR
E. GOOD --- MAGAZINE WITH 50-60% OF READERS HAVING ANY COLLEGE IS GOOD
E. POOR --- MAGAZINE WITH UNDER 40% READERS HAVING ANY COLLEGE IS POOR
ED CONT --- EDITORIAL CONTENT
ED EXCL --- EXCELLENT EDITORIAL CONTENT MATCH
ED FAIR --- FAIR EDITORIAL CONTENT MATCH
ED GOOD --- GOOD EDITORIAL CONTENT MATCH
ED POOR --- POOR EDITORIAL CONTENT MATCH
EDUCAT'N --- % READERSHIP WITH SOME COLLEGE EDUCATION
H. EXCEL --- MAGAZINE WITH >60% OF HOME READERS IS EXCELLENT
H. FAIR --- MAGAZINE WITH 40-50% OF HOME READERS IS FAIR
H. GOOD --- MAGAZINE WITH 50-60% OF HOME READERS IS GOOD
I. EXCEL --- MAGAZINE WITH >20% OF READERS HAVING INCOME >$35,000 IS EXCELLENT
I. FAIR --- MAGAZINE WITH 10-15% OF READERS HAVING INCOME >$35,000 IS FAIR
I. GOOD --- MAGAZINE WITH 15-20% OF READERS HAVING INCOME >$35,000 IS GOOD
I. LOW --- MAGAZINE WITH LESS THAN 10% READERS HAVING INCOME >$35,000 IS LOW
IN HOME --- % IN HOME READERSHIP
INCOME --- % READERSHIP EARNING MORE THAN $30,000 PER YEAR

The pairwise comparison process yields priorities for the ratings as shown above. The global priorities are used when rating the magazines. Each magazine is given a rating with respect to each criterion. For example, with respect to the Income criterion, a magazine rated as Excellent would have .073 added to its effectiveness index, while a magazine rated Low would receive a value of .008. The ratings and total effectiveness for each magazines are shown below.

ALTERNATIVES	INCOME .1482	EDUCAT'N .0425	AGE .1577	CAMERA B .4404	IN HOME .1557	ED CONT .0557	TOTAL
1 NATL GEOGRAPHIC	I. GOOD	E. GOOD	A. GOOD	C. GOOD	H. EXCEL	ED GOOD	0.311
2 NEWSWEEK	I. GOOD	E. GOOD	A. EXCEL	C. GOOD	H. FAIR	ED EXCL	0.331
3 SOUTHERN LIVING	I. LOW	E. FAIR	A. FAIR	C. GOOD	H. EXCEL	ED POOR	0.243
4 PEOPLE	I. LOW	E. FAIR	A. EXCEL	C. GOOD	H. FAIR	ED POOR	0.260
5 SPORTS ILLUS.	I. GOOD	E. FAIR	A. EXCEL	C. GOOD	H. GOOD	ED EXCL	0.346
6 TRAVEL & LEISURE	I. EXCEL	E. EXCEL	A. FAIR	C. EXCEL	H. FAIR	ED POOR	0.388
7 TIME	I. FAIR	E. GOOD	A. EXCEL	C. GOOD	H. GOOD	ED EXCL	0.327
8 U.S. NEWS	I. GOOD	E. GOOD	A. GOOD	C. EXCEL	H. FAIR	ED EXCL	0.402

Next we must consider which combination of alternatives is "best", subject to constraints. Suppose our only constraint is budget. If we know the budgetary requirements of each of the alternatives, we can formulate an integer linear programming model as follows:

```
Maximize  E1 R1 X1  + E2 R2 X2  +   .... + E8 R8 X8,

  where
    Magazine 1 is National Geographic,
    R1 = 21,051 (the number of readers)
    E1 = .311 (the effectiveness coefficient from the Ratings model),
    X1 will be determined and will be 1 if it is optimal to
       advertise in National Geographic, 0 otherwise,

    Magazine 2 is Newsweek
    R2 = 15,594 (the number of readers)
    E2 = .331 (the effectiveness coefficient from the Ratings model),
    X2 will be determined and will be 1 if it is optimal to
       advertise in Newsweek, 0 otherwise,
  .
  .
  .
    Magazine 8 is U.S. News
    R8 =  8,929 (the number of readers)
    E8 = .402 (the effectiveness coefficient from the Ratings model),
    X8 will be determined and will be 1 if it is optimal to
       advertise in U. S. News, 0 otherwise.

Subject to the constraint on the total advertising budget:

    346,080 X1 + 780,180 X2 + 11,370 X3 + 605,880 X4 + 965,940 X5 +
               183,216 X6 + 1,324,282 X7 + 100,740 X8 <= $1,500,000
```

The final integer programming (IP) solution is:

```
X1 (NATL GEOGRAPHIC) = 1
X2 (NEWSWEEK)        = 0
X3 (SOUTHERN LIVING) = 1
X4 (PEOPLE)          = 1
X5 (SPORTS ILLUS.)   = 0
X6 (TRAVEL & LEISURE)= 1
X7 (TIME)            = 0
X8 (U.S. NEWS)       = 1
```

or, in other words, the decision should be to advertise in National Geographic, Southern Living, People, Travel & Leisure, and U.S. News magazines.

The "optimal" solution from the ILP formulation should not be taken as the final decision. Rather, it must be examined to see if it suggests other criteria that should be added to the AHP formulation, and/or a change in judgments in the AHP model, and/or additional constraints for the ILP model. Iteration is performed until an acceptable, "optimal" solution is achieved.

The same approach can be used to determine the best combination of R&D projects for a company. Criteria such as market position, fit with strategic direction, and projected sales can be used in the AHP model. Constraints that preclude too much redundancy or require a minimum amount of research in a given area can easily be included in the ILP model.

Care must be taken to assure that the decision truly reflects managements objectives and constraints. Not only can a piecemeal analysis be difficult to synthesize into the decision process, but the results prove to be troublesome. As an example, a Fortune 500 company recently used AHP to rate R&D projects. They were satisfied with both the process of arriving at the priorities and the priorities themselves. However, they did not think through the resource allocation problem thoroughly and simply allocated funds from their budget to the projects in rank order until no more funding remained. This resulted in some obvious weaknesses. Some departments got very large increases in funding while others got very large decreases. The departments with large increases were happy and quiet. The departments with large decreases were unhappy and very vocal. Something was wrong with the process! Furthermore, it appeared that some research areas had an overabundance of funding while others had too little funding. With a little bit more thought about objectives and constraints, the resource allocation could have been greatly improved.

One objective of the organization was to keep their employee morale high. Employee morale in those departments with large reductions in funding suffered. Management could have included constraints in the ILP formulation that guaranteed a somewhat smoother transition from the present R&D funding to a more desirable one. For example, constraints that guaranteed that each department get at least a given percentage of the previous years allocation would have prevented any drastic changes that adversely effected employee morale. Other constraints to guarantee that a minimum amount of diversification and a minimum amount of coverage to specific research areas could easily be accommodated. Thus, with a little thought about the objectives and constraints, and with some iteration, the AHP/ILP combination is a powerful mechanism for allocating resources so as to "best" meet an organizations objectives.

CONCLUSION

A likely reason that so few decision makers use OR/MS techniques today in making decisions is the inability for them to synthesize the quantitative OR/MS results with the qualitative factors that they have learned from experience to be so important. The Analytic Hierarchy Process is an ideal synthesizing vehicle that can help decision makers integrate information from OR/MS studies into the decision process. If we can show decision makers how OR/MS tools can support *their* decision making, in *their* context of spreadsheets and pro/con analysis, and help *them* synthesize all the available information according to *their* view of the world, then we are bound to increase the utilization and impact of existing OR/MS methodologies.

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