AN APPROACH TO MULTIATTRIBUTE INTERVAL-VALUED INTUITIONISTIC FUZZY GROUP DECISION MAKING BASED ON STOCHASTIC DEMATEL, FUZZY ANP, AND IDEAL POINT.

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ABSTRACT

The purpose of this paper is to develop a multiattribute interval-valued intuitionistic fuzzy group decision making by using ranking index based on particular measure of closeness to the positive ideal solution (PIS) and using Hamming distance to measure differences between each alternative and the PIS as well as the negative ideal solution (NIS).The main idea of this methodology is introduced by (Li, Huang, and Chen, 2010) by independently considering a number of criteria. However, in real world, there are dependent relationships among criteria and sometimes there is uncertain information. So that, we utilized a stochastic decision making trial and evaluation laboratory (SDEMATEL), a fuzzy analytical network process (FANP) to determine the weights of the attributes and to develop previous model, we used interval-valued intuitionistic fuzzy set as entrance information.

Keywords: group decision making, interval-valued intuitionistic fuzzy set, Stochastic DEMATEL, Fuzzy ANP.

1. Introduction

Multiattribute group decision making is so important since, it's used in so many fields such as management, operation research and so on. So we tried to represent an efficient model to improve decision making in a group. The main idea of the proposed model is introduced by (Li, Huang, and Chen, 2010).Their model is efficient because it aggregates different kinds of information including multi-granularity linguistic labels, fuzzy numbers, interval numbers and real numbers. To develop this model, we consider a new kind of information called interval-valued intuitionistic fuzzy set as entrance information. Moreover, in real world, there are interrelations among the criteria and there are imprecise, vague, and uncertain environments. So that, we use an aggregated method of Stochastic DEMATEL and Fuzzy ANP to determine the weights of the criteria.

In next section, first by using stochastic DEMATEL, we extract interrelations among criteria and divide them to a cause and effect group to plot a network relationship map. Then, we use Fuzzy ANP to determine the weights of criteria in a fuzzy environment. After, we introduce the main algorithm to

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handle multiattribute group decision making by interval-valued intuitionistic fuzzy set as entrance information. Finally, in section 3 and 4, we express advantages and validity of proposed model and present suggestions for future works.

2. Procedure for multiattribute interval-valued intuitionistic fuzzy group decision making

Here, we use stochastic DEMATEL to deal with interrelations among each criterion. Then, by using fuzzy ANP, the weights of each criterion are calculated. Finally, the rank of alternatives is determined by particular measure of closeness to positive ideal solution (PIS).

2.1.Stochastic DEMATEL

Decision Making Trial and Evaluation Laboratory (DEMATEL) method enables us to extract interrelationship among factors in a complex problem. Since, it's based on graph theory, by visualizing problem, it helps us divide multiple criteria into a cause-and-effect group. So that, we can plot a network relationship map (NRM) to understand causal relationships better. However, in the case of existing uncertainty in the problem, the ordinary DEMATEL can't deal with flexible interrelationship among factors. So, stochastic DEMATEL can handle it on the various situations (Tamura, and Akazawa, 2005).The stochastic DEMATEL method can be summarized into following steps (Liou et al., 2007; Tamura, and Akazawa, 2005):

Step1: construct the average matrix X^s(also called initial direct relation matrix). Suppose we have m factors and k experts to consider. Since the elements of the direct influence matrix is uncertain, expectation and variance of probability distribution is obtained by the dispersion of the data contained in multiple respondents reply in the direct matrix. The direct matrix for each expert is obtained by asking them to indicate the degree which represents his or her idea about the effect of criterion i on criterion j. For instance, we can use a 5-grade evaluation and then we normalize these direct matrices. Probability density function is assumed to be a cutting normal distribution on [0, ∞).based on these probabilistic information numerous stochastic matrices are generated by a Monte Carlo method. Afterwards, based on being neutral, pessimistic, or optimistic we use expected value, 2.5 percentile and 97.5 percentile, respectively and we construct the average matrix as follows:

$$X^s = [x_{ij}]_{m\times m} \tag{1}$$

Step2: Normalize the initial direct relation matrix. The normalized stochastic direct matrix is obtained as follows:

$$X_r^s = \lambda \cdot X^s \tag{2}$$

$$\text{where} \quad \lambda = 1/\text{the largest row sum of } X^s \tag{3}$$

Step3: Compute the total relation matrix as follows:

$$X^{sf} = X_r^s + (X_r^s)^2 + \cdots = X_r^s(I - X_r^s)^{-1} \tag{4}$$

This matrix is a stochastic direct/indirect matrix. We also define $\mathbf{r}$ and $\mathbf{c}$ vectors representing the sum of rows and sum of columns of the total relation matrix as follows:

$$\mathbf{r} = [r_i]_{m\times 1} = \left(\sum_{j=1}^{m} x_{ij}^{sf}\right)_{m\times 1} \tag{5}$$

$$c = [c_i]'_{1\times m} = \left(\sum_{i=1}^{m} x_{ij}^{sf}\right)'_{1\times m} \tag{6}$$

Where superscript $'$ denotes transpose. $r_i = \sum_{j=1}^{m} x_{ij}$ shows the total effects, given by criterion i to the other criteria $j = 1, \dots, m$, both directly and indirectly. $c_i = \sum_{i=1}^{m} x_{ij}$ shows the total effects, received by criterion j from the other criteria $i = 1, \dots, m$, both directly and indirectly.

Step4: Set a threshold value p and obtain the network relationship map. The criteria whose effect in the total relation matrix is greater than the threshold value should be considered and shown in a network relationship map and the values of elements in matrix X^{sf} are zero if their values less than p. Then, a new total-influence matrix X_p^{sf} can be obtained. It should be mentioned that based on being neutral, pessimistic, or optimistic a threshold value would be different.

$$X_p^{sf} = [x_{ij}^{p}]_{m\times m} \tag{7}$$

2.2 Fuzzy ANP

In order to better understand the Fuzzy ANP, first, we introduce ANP and then, Fuzzy ANP will be expressed.

2.2.1ANP

The ANP is an extension of AHP and it enables us to consider dependence within a criterion (inner dependence) and among different criteria (outer dependence).The ANP allows feedback relationship among criteria.

The first phase of ANP is forming a supermatrix which compares the measuring criteria in overall system. The relative importance of pair-wise comparisons can be categorized from 1 to 9 which represent pairs of equal importance (1) to extreme inequality in importance (9) (Saaty, 1980). The general form of supermatrix can be formed as (Liou, Tzeng, and Chang, 2007):

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1m} \\ W_{21} & W_{22} & \cdots & W_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1} & W_{m2} & \cdots & W_{mm} \end{bmatrix} \tag{8}$$

Where c_m denotes the mth cluster, e_{ml} denotes the lth element in mth cluster, and W_{ij} is the principal eigenvector of the influence of the elements compared in the jth cluster to the ith cluster.

After forming supermatrix, the weighted supermatrix is generated by transforming all columns sums to unity (Ong, Huang, and Tzeng, 2004).then the weighted supermatrix w_w will be raised to limitilng powers l to obtain global weights as follows:

$$\lim_{l\to\infty} W_w^{l} \tag{9}$$

If there is more than one limiting super matrix, the final weighted limiting matrix can be calculated as follows (e.g. the average priority weights):

$$\lim_{k\to\infty}\left(\frac{1}{N}\right)\sum_{j=1}^{N} W_j^k \tag{10}$$

Where $W_j$denoted the jth limiting supermatrix.

2.2.2Fuzzy ANP

Fuzzy ANP has been represented before by (Chen, and Chen, 2009). In their model, they used DEMATEL just to plot network relationship map and they obtained weighted matrix by considering NRM. We adopt the hybrid model represented by (Yang, and Tzeng, 2010) to uncertain and fuzzy environment.

Fuzzy ANP can deal with unspecific and fuzzy characteristics. The steps of the fuzzy ANP can be summarized as follows:

Step1: construct supermatrix which it can be done through pair-wise comparisons among criteria by asking "how much importance does a criterion have compared to another criterion?"

The relative importance value can be given through table 1.

Table 1

Fuzzy number	Linguistic variable	Triangular fuzzy number
$\tilde{9}$	Extremely important/preferred	(7,9,9)
$\tilde{7}$	Very strongly important/preferred	(5,7,9)
$\tilde{5}$	Strongly important/preferred	(3,5,7)
$\tilde{3}$	Moderately important/preferred	(1,3,5)
$\tilde{1}$	Equally important/preferred	(1,1,3)

To obtain the elements of the supermatrix, determine the local weights of criteria by utilizing pair-wise comparison matrices. For example if we have a system with four criteria, the local weights of criteria 2 through 4 under the effect of criterion 1 will be obtained as follows (Chen, and Chen, 2009):

Table 2

Measurement criteria	C2	C3	C4	Local weight
C2	(1,1,3)	(1.91,3.31,4.27)	(0.14,0.18,0.31)	0.22
C3	(0.23,0.3,0.52)	(1,1,3)	(0.19,0.28,0.49)	0.12
C4	(3.18,5.66,7.10)	(2.02,3.60,5.13)	(1,1,3)	0.66

The general form of supermatrix can be formed as Eq.8

Step2: The weighted supermatrix W_w such as Eq. (13) can be calculated by multiplying the unweighted supermatrix and the normalized total-influence matrix X_z^{sf}.Total-influence matrix, which is derived according to DEMATEL method, can be normalized as follows:

$$X_z^{sf} = \begin{bmatrix} x_{11}^z & \cdots & x_{1j}^z & \cdots & x_{1m}^z \\ \vdots & & \vdots & & \vdots \\ x_{i1}^z & \cdots & x_{ij}^z & \cdots & x_{im}^z \\ \vdots & & \vdots & & \vdots \\ x_{m1}^z & \cdots & x_{mj}^z & \cdots & x_{mm}^z \end{bmatrix} \tag{11}$$

$$where\ x^{z}_{ij} = \frac{x^{p}_{ij}}{\sum_{i=1}^{m} x^{p}_{ij}} \tag{12}$$

The weighted supermatrix W_w can be obtained as follows:

$$W_w = X_z^{sf} \times W \tag{13}$$

Step3: raise the weighted supermatrix to limiting powers l to get the global weights by Eq. (9). If there is more than one limiting super matrix, the final weighted limiting matrix can be calculated by Eq. (10).

2.2.3Numerical example

The numbers of this example is extracted from (Ou Yang, Shieh, Leu and Tzeng, 2008).Suppose that we have six criteria which can be divided to two clusters. The NRM and total influence matrix are obtained by stochastic DEMATEL as follows:

Table 3.The total influence matrix

	Cluster 1	Cluster 2
Cluster 1	0.1	3
Cluster 2	0.4	0.1

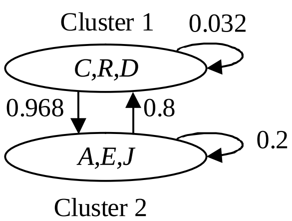

Figure 1.The NRM of relations

The normalized total influence matrix is as follows:

Table 4.The normalized total influence matrix

	Cluster 1	Cluster 2
Cluster 1	0.032	0.968
Cluster 2	0.800	0.200

The unweighted supermatrix can be calculated through the step 1 as follows:

$$\begin{array}{c c} & \begin{array}{cccccc} C & R & D & A & E & J \end{array} \\ \begin{array}{c} C \\ R \\ D \\ A \\ E \\ J \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0.634 & 0.25 & 0.4 \\ 0 & 1 & 0 & 0.192 & 0.25 & 0.2 \\ 0 & 0 & 1 & 0.174 & 0.5 & 0.4 \\ 0.637 & 0.582 & 0.105 & 1 & 0 & 0 \\ 0.105 & 0.109 & 0.637 & 0 & 1 & 0 \\ 0.259 & 0.309 & 0.258 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The weighted super matrix is obtained by Eq. (13) as follows:

$$W_w^{tra} = \begin{array}{c} \\ C \\ R \\ D \\ A \\ E \\ J \end{array} \begin{array}{c} \begin{array}{cccccc} C & R & D & A & E & J \end{array} \\ \begin{bmatrix} 0.500 & 0.000 & 0.000 & 0.317 & 0.125 & 0.200 \\ 0.000 & 0.500 & 0.000 & 0.096 & 0.125 & 0.100 \\ 0.000 & 0.000 & 0.500 & 0.087 & 0.250 & 0.200 \\ 0.318 & 0.291 & 0.053 & 0.500 & 0.000 & 0.000 \\ 0.052 & 0.055 & 0.319 & 0.000 & 0.500 & 0.000 \\ 0.129 & 0.155 & 0.129 & 0.000 & 0.000 & 0.500 \end{bmatrix} \end{array}$$

Finally the global weights are obtained through the step 3 as follows:

$$W_f^{tra} = \begin{array}{c} \\ C \\ R \\ D \\ A \\ E \\ J \end{array} \begin{array}{c} \begin{array}{cccccc} C & R & D & A & E & J \end{array} \\ \begin{bmatrix} 0.232 & 0.232 & 0.232 & 0.232 & 0.232 & 0.232 \\ 0.105 & 0.105 & 0.105 & 0.105 & 0.105 & 0.105 \\ 0.163 & 0.163 & 0.163 & 0.163 & 0.163 & 0.163 \\ 0.226 & 0.226 & 0.226 & 0.226 & 0.226 & 0.226 \\ 0.140 & 0.140 & 0.140 & 0.140 & 0.140 & 0.140 \\ 0.135 & 0.135 & 0.135 & 0.135 & 0.135 & 0.135 \end{bmatrix} \end{array}$$

2.3 The multiattribute interval-valued intuitionistic fuzzy group decision making algorithm based on ideal point

The main idea of proposed model represented by (Li, Huang, and Chen, 2010) and we obtain the weights of criteria by an integrated method which was described in two last sections to deal with flexibility and feedback among the criteria. First, we introduce interval-valued intuitionistic fuzzy set in a definition proposed by (Atanassov, and Gargov, 1989) as follows:

Definition: let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse, $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$ .an interval-valued intuitionistic fuzzy set A in X is an expression given by

$$A = \{\langle x, t_A(x_i), f_A(x_i)\rangle | x_i \in X\} \tag{14}$$

Where $t_A: X \to D[0,1], f_A: X \to D[0,1]$ with the condition $0 \le \sup t_A(x_i) + \sup f_A(x_i) \le 1$.the intervals $t_A(x_i)$ and $f_A(x_i)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x_i to the set A.

For any two intervals $[a,b]$and $[c,d]$ with $b + d \le 1$ belonging to $D[0,1]$, let $t_A(x) = [a,b], f_A(x) = [c,d]$,so an interval-valued intuitionistic fuzzy set whose value is denoted by $A = \{\langle x, t_A(x_i), f_A(x_i)\rangle | x_i \in X\}$ .in this paper we call $([a,b],[c,d])$ an interval-valued intuitionistic value.

The steps of the proposed algorithm can be summarized as follows:

Step1: construct PIS and NIS for the decision makers. Suppose we have n alternatives and m attributes to consider. Let the PISs and NISs of the decision makers e_k $(k = 1,2, \dots, K)$ be x^{k+} and x^{k-}, respectively; whose rating vectors are denoted by $y^{k+} = (y_1^{k+}, y_2^{k+}, \dots, y_m^{k+})$, and $y^{k-} = (y_1^{k-}, y_2^{k-}, \dots, y_m^{k-})$ respectively; and O be the set of attributes, where

$$y_i^{k+} = ([a_i^{k+}, b_i^{k+}], [c_i^{k-}, d_i^{k-}]) \quad i = 1, \dots, m \tag{15}$$

$$\begin{cases} ([a_i^{k+}, b_i^{k+}], [c_i^{k-}, d_i^{k-}]) = ([\max_j a_{ij}^k, \max_j b_{ij}^k], [\min_j c_{ij}^k, \min_j d_{ij}^k]) & j = 1, \dots, n; i \in O^b \\ ([a_i^{k+}, b_i^{k+}], [c_i^{k-}, d_i^{k-}]) = ([\min_j a_{ij}^k, \min_j b_{ij}^k], [\max_j c_{ij}^k, \max_j d_{ij}^k]) & j = 1, \dots, n; i \in O^c \end{cases} \tag{16}$$

O^b is the set of benefit attributes. O^c is the set of cost attributes.

$$y_i^{k-} = ([a_i^{k-}, b_i^{k-}], [c_i^{k+}, d_i^{k+}]) \quad i = 1, \ldots, m \tag{17}$$

$$\begin{cases} ([a_i^{k-}, b_i^{k-}], [c_i^{k+}, d_i^{k+}]) = ([\min_j a_{ij}^k, \min_j b_{ij}^k], [\max_j c_{ij}^k, \max_j d_{ij}^k]) \; j = 1, \ldots, n; i \in O^b \\ ([a_i^{k-}, b_i^{k-}], [c_i^{k+}, d_i^{k+}]) = ([\max_j a_{ij}^k, \max_j b_{ij}^k], [\min_j c_{ij}^k, \min_j d_{ij}^k]) \; j = 1, \ldots, n; i \in O^c \end{cases} \tag{18}$$

Step2: determine the weights of the attributes for the group by using SDEMATEL method and fuzzy ANP. Denote weights of the attributes $i = 1, \ldots, m$ for the group by W^G.the W^G will be obtained by Eq. (13).

Step3: compute the distances between each alternative and the PIS as well as the NIS for the decision makers.

$$d(y_{ij}^k, y_i^{k+}) = \frac{1}{4} \sum_{i=1}^m W_i^G (|a_{ij}^k - a_i^{k+}| + |b_{ij}^k - b_i^{k+}| + |c_{ij}^k - c_i^{k+}| + |d_{ij}^k - d_i^{k+}|) j = 1, \ldots, n \tag{19}$$

$$d(y_{ij}^k, y_i^{k-}) = \frac{1}{4} \sum_{i=1}^m W_i^G (|a_{ij}^k - a_i^{k-}| + |b_{ij}^k - b_i^{k-}| + |c_{ij}^k - c_i^{k-}| + |d_{ij}^k - d_i^{k-}|) j = 1, \ldots, n \tag{20}$$

Step4: compute relative closeness degrees of alternatives to the PISs for the decision makers.

$$\begin{cases} d^-(y^{k+}) = max\{d(y_j^k, y^{k+}) | j = 1,2, \ldots, n\} \\ d^+(y^{k+}) = min\{d(y_j^k, y^{k+}) | j = 1,2, \ldots, n\} \end{cases}$$
(21)

$$\begin{cases} d^+(y^{k-}) = max\{d(y_j^k, y^{k-}) | j = 1,2, \ldots, n\} \\ d^-(y^{k-}) = min\{d(y_j^k, y^{k-}) | j = 1,2, \ldots, n\} \end{cases}$$
(22)

$$\tau(y_j^k) = \varepsilon^k \times \frac{d^-(y^{k+}) - d(y_j^k, y^{k+})}{d^-(y^{k+}) - d^+(y^{k+})} + (1 - \varepsilon^k) \times \frac{d(y_j^k, y^{k-}) - d^-(y^{k-})}{d^+(y^{k+}) - d^-(y^{k-})} \tag{23}$$

Where the parameters $\varepsilon^k \in [0,1]$ are compromise coefficients, which may be regarded as decision making strategy "the majority of attributes" closing to the PIS, x^{K+} (Li, 2007). Denote $\tau(y_j^k)$ by τ_j^k.then, construct a matrix which expresses relative closeness degrees of alternatives x_j $(j = 1,2, \ldots, n)$ for all decision makers e_k $(k = 1,2, \ldots, K)$ as follows:

$$\tau = (\tau_j^k)_{k \times n} \tag{24}$$

Step5: compute weights of the decision makers. A moderator gives an expertise point in range [0-100] for each expert. Then, the weights of expertise are calculated as follows:

$$W_{ei} = \frac{P_{ei}}{\sum_{i=1}^z P_{ei}}, i = 1,2, \ldots, z \tag{25}$$
Where e_i and p are ith expert and expertise point, respectively. We denote W_{ei} by W^k.

Step6: compute relative closeness degrees of alternatives with respect to the PISs for the group. The decision makers are regarded as "attributes". So that, decision making problem with decision matrix τ given by Eq. (24) may be regarded an MADM problem with n alternatives x assessed on k attributes (e.g.

decision makers). The PIS and the NIS for the group can be defined as x^+ and x^-, whose vectors are denoted by $\tau^+ = (\tau_1^+, \tau_2^+, \dots, \tau_K^+)^T$ and $\tau^- = (\tau_1^-, \tau_2^-, \dots, \tau_K^-)^T$, respectively, where

$$\begin{cases} \tau_k^+ = max\{\tau_j^k | j = 1,2,\dots,n\}, \ (k = 1,2,\dots,K) \\ \tau_k^- = min\{\tau_j^k | j = 1,2,\dots,n\}, \ (k = 1,2,\dots,K) \end{cases} \tag{26}$$

The distances (p-power of the weighted Minkowski distance) between an alternative $x_j (j = 1,2,..,n)$ and the PIS x^+ as well as the NIS x^- for the group are defined as follow:

$$\begin{cases} \rho_p^G(x_j, x^+) = \sqrt[p]{\sum_{k=1}^K [w^k(\tau_k^+ - \tau_j^k]^p} \\ \rho_p^G(x_j, x^-) = \sqrt[p]{\sum_{k=1}^K [w^k(\tau_k^- - \tau_j^k]^p} \end{cases} \tag{27}$$

Let

$$\begin{cases} \rho_p^{G-}(x^{k+}) = max\{\rho_p^G(x_j^k, x^{k+}) | j = 1,2,\dots,n\} \\ \rho_p^{G+}(x^{k+}) = min\{\rho_p^G(x_j^k, x^{k+}) | j = 1,2,\dots,n\} \end{cases} \tag{28}$$

$$\begin{cases} \rho_p^{G+}(x^{k-}) = max\{\rho_p^G(x_j^k, x^{k-}) | j = 1,2,\dots,n\} \\ \rho_p^{G-}(x^{k-}) = min\{\rho_p^G(x_j^k, x^{k-}) | j = 1,2,\dots,n\} \end{cases} \tag{29}$$

$$\tau_p^G(x_j) = \varepsilon^k \times \frac{\rho_p^{G-}(x^{k+}) - \rho_p^G(x_j^k, x^{k+})}{\rho_p^{G-}(x^{k+}) - \rho_p^{G+}(x^{k+})} + (1 - \varepsilon^k) \times \frac{\rho_p^G(x_j^k, x^{k-}) - \rho_p^{G-}(x^{k-})}{\rho_p^{G+}(x^{k-}) - \rho_p^{G-}(x^{k-})} \tag{30}$$

Where the parameters $\varepsilon^k \in [0,1]$ are compromise coefficients, which may be regarded as decision making strategy "maximum group utility". Apparently, $0 \le \tau_p^G(x_j) \le 1$.the larger $\tau_p^G(x_j)$ the better alternatives x_j (j=1,2,…,n) for the group. We can rank the alternatives according to the non increasing order of all values $\tau_p^G(x_j)$ and the best alternative for the group is the one with largest $\tau_p^G(x_j)$.

3. Discussion

As it was mentioned, the framework of this model is represented by (Li, Huang, and Chen, 2010) and the main difference between these two models is the way of determining the weights. In our model, we use SDEMATEL and FANP to calculate weights by considering feedbacks among attributes in a fuzzy environment that is closer to the real world. Using DEMATEL and ANP to determine the weights of attributes has been implemented in many articles such as (Chen, and Chen, 2010) which used this method to determine the weights in a TOPSIS procedure. Since our approach is based on ideal points, using this method to determine the weights is completely logical. In addition, the privilege of our approach to fuzzy TOPSIS is that it can deal with any kind of heterogeneous information which is more suitable.

3. Conclusion

This study presents a multi-attribute interval-valued intuitionistic fuzzy group decision making using ranking index to PIS, where the interrelations among the criteria are considered and we use stochastic DEMATEL to plot NRM and then we use fuzzy ANP to determine the weights of the attributes. In comparison with previous method, our model can handle interrelations among the criteria and by using stochastic DEMATEL we can consider different degrees of influence among the criteria in constructing the supermatrix in an uncertain environment.

For future work, we can use this algorithm in real world empirical situations. And we can plan a group decision support system based on this algorithm.

REFERENCES

Atanassov, K., & Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31*, 343-349.

Chen, J.-K., & Chen, I.-S., (2010). Using a novel conjunctive MCDM approach based on DEMATEL, fuzzy ANP, and TOPSIS as an innovation support system for Taiwanese higher education. *Expert Systems With Applications, 37*(3), 1981-1990.

Li, D.-F. (2007). Compromise ratio method for fuzzy multi-attribute group decision making. *Applied Soft Computing, 7*, 807-817.

Li, D.-F., & Huang, Z.-G., & Chen, G.-H. (2010). A systematic approach to heterogeneous multiattribute group decision making. *Computers & Industrial Engineering, 59(4),* 561-572. doi:10.1016/j.cie.2010.06.015

Liou, J.H., Tzeng, G.H., & Chang, H.C. (2007). Airline safety measurement using a hybrid model. *Journal of Air Transport Management, 13,* 243-249.

Ong, C. S., Huang, J. J., & Tzeng, G. H. (2004). Multidimensional data in multidimensional scaling using the analytic network process. Pattern Recognition Letters, 26(6), 755–767.

Ou Yang, Y.-P., Shieh, M.-H., Leu, J.-D., & Tzeng G.-H., (2008).A novel hybrid MCDM model combined with DEMATEL and ANP with applications. *International Journal of Operations Research, 5 (3),* 160-168 .

Saaty, T. L. (1980). The analytic hierarchy process. NY: McGraw-Hill.

Tamura, H., & Akazawa, K., (2005). Stochastic DEMATEL for structural modeling of a complex problematique for realizing safe, secure and reliable society. *Journal of telecommunications and information technology, 4,* 139-146.

Yang, J.L., & Tzeng, G.H., (2010). An integrated MCDM technique combined with DEMATEL for a novel cluster-weighted with ANP method. *Expert Systems with Applications, 38(3)*, 1417-1424.