Determining a support site for internally displaced pupils by extending the EVAMIX method to collective decision-making based on the Choquet and Sugeno integral in Burkina Faso.

Hadarou YIOGO[1], Zoïnabo SAVADOGO[2]
[1]Joseph KI-ZERBO University,UFR-SEA/LANIBIO, Burkina Faso.E-mail: yiogohadarou95@gmail.com
[2]Joseph KI-ZERBO University,UFR-SEA/LANIBIO, Burkina Faso. E-mail: serezenab@yahoo.fr

Abstract

The security situation in Burkina Faso has deteriorated in recent years. This has had a negative impact on the country's education system through the closure of several schools. Given the importance attached to education, it is necessary to set up reception sites for displaced pupils in order to ensure the continuity of their studies. There are a number of methods available to aid decision-making, but some of them are not without their shortcomings. In this work, we address the problem by extending the EVAMIX method to group decision-making using the Choquet and Sugeno integral, taking into account the order of importance of the decision-makers, and applying it to the problem of identifying the best site. Through a simulation, we applied our method to four sites and obtained good results.

Key words: Collective decision, extension-EVAMIX, Choquet integral, Sugeno integral, site determination.

1 Introduction

Burkina Faso has been facing a situation of insecurity for several years. This has led to instability in national education. In view of this disastrous phenomenon, it is imperative to accommodate these schoolchildren in order to facilitate the continuation of their learning. Several sites have been identified for these activities, but the question is: which site should be chosen to accommodate these pupils? This question can be answered using multi-criteria decision support. Numerous methods exist in the literature, but some of them are not without criticism [1]. The EVAMIX method is a single-decision multi-criteria decision support method with satisfactory properties [2]. Nowadays, many decision problems require that the decision is not taken by a single person but rather by a group of people [3]. This is why the present work aims at improving the decision support method. This is why the present work aims to propose an extension of the EVAMIX method to collective decision-making by integrating aggregation operators such as the Choquet and Sugeno integral using a different importance of the decision-makers and applying it to the problem of choosing the best site. After a brief presentation of the literature review, our extension and its application to the choice of the best site will follow.

2 Literature review

2.1 Choquet integral

According to [4], for a capacity ν, the Choquet integral of an action a_i represented by a vector of $\mathbb{R}^m$ is defined by:

$$C_\nu(a_i) = \sum_{j=1}^{m} \left[f_{\sigma(j)}(a_i) - f_{\sigma(j-1)}(a_i)\right] \nu(\mathbb{A}_{\sigma(j)})$$

where σ is the permutation which reorders the performances of a_i in ascending order, that is $f_{\sigma(1)}(a_i) \leq f_{\sigma(2)}(a_i) \leq ... \leq f_{\sigma(m)}(a_i)$ with $f_{\sigma(0)}(a_i) = 0$. with $\mathbb{A}_{\sigma(j)} = \left\{f_{\sigma(j)}, f_{\sigma(j+1)}, ..., f_{\sigma(m)}\right\}$ and $\mathbb{A}_{\sigma(m+1)} = \{\}$.

2.2 Sugeno integral

According to [4], the discrete Sugeno integral is a function of the following form:

$$S_\mu(x) = \max_{1\leq k\leq n} \left[\min\left(x_{(k)}; \mu(\{(k), ..., (n)\})\right)\right] \, (x \in [0;1]^n)$$

where μ is a fuzzy measure on N i.e. a monotone set function $\mu : 2^N \longrightarrow [0;1]$ verifying $\mu(\emptyset) = 0$ and $\mu(N) = 1$. Moreover (.) represents a permutation on N such that $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$

2.3 Lorentz (mean) combination

The Lorentz mean of n x_k values is defined by:

$$L_{1/3}(x) = \left(\frac{\sum_{k=1}^{n} x_k^{1/3}}{n}\right)^3 \tag{1}$$

2.4 Description de la méthode EVAMIX

According to [5], the EVAMIX method was developed by VOOGD in 1983 and refers to mixed evaluations (qualitative and quantitative). To compare one action with another, we first calculate two dominance indices, the first for qualitative evaluations and the second for quantitative evaluations. These two indices are then normalised and combined to give an overall measure of dominance. Finally, the overall score for each action is calculated and this will result in a ranking of the actions from best to worst.

3 Principle of the extension of the EVAMIX method based on the Choquet and Sugeno integrals(EMEBICS).

Step 1: Calculating the weight of each decider we define the set $N = \{N_1, ..., N_K\}$ with N_k the number of years of experience of decision-maker k.

$$\mu(\{k\}) = \frac{N_k}{\sum_{k=1}^{K} N_k} \quad et \quad \mu(\{k\}, \{k+1\}, ..., \{K\}) = \mu(\{k\}) + \mu(\{k+1\}) + ... + \mu(\{K\})$$

Step 2: Global weight per criterion of the k decision-makers

$$\begin{cases} P_j = \left(\dfrac{\sum_{k=1}^{K} (p_j^k)^{1/3}}{n}\right)^3 ,; \quad j = 1, ..., M \\ P = \{p_1, p_1, ..., p_M\} \end{cases} \tag{2}$$

With p_j^k the weight assigned to criterion g_j by decision-maker k.

Step 3: Calculation of the dominance index of the actions α_{ij}^l and β_{ij}^l for a given decision-maker d_l.

$$\begin{cases} \alpha_{ii'}^l = \left[\sum_{k\in O} \left\{p_k^l \times sgn(g_k^l(a_i) - g_k^l(a_j))\right\}^c\right]^{1/c} \\ \beta_{ij}^l = \left[\sum_{\in C} \{p_k^l \times (g_k^l(a_i) - g_k^l(a_j))\}^c\right]^{1/c} \\ i, j \in \{1, ..., N\}, k \in \{1, ..., M\}, l \in \{1, ..., S\}, c = 1 \end{cases} \quad with \quad sgn(g_k^l(a_i) - g_k^l(a_j)) = \begin{cases} -1 \quad si \quad g_k^l(a_i) < g_k^l(a_j) \\ 0 \quad si \quad g_k^l(a_i) \simeq g_k^l(a_j) \\ 1 \quad si \quad g_k^l(a_i) > g_k^l(a_j) \end{cases} \tag{3}$$

Step 4: Normalise the dominance index α_{ij}^{l} and β_{ij}^{l} of the actions

$$\begin{cases} \delta_{ij}^{l} = \frac{(\alpha_{ij}^{l}-\alpha^{-})}{(\alpha^{+}-\alpha^{-})} \quad (attribues \quad ordinales) \\ d_{ij}^{l} = \frac{(\beta_{ij}^{l}-\beta^{-})}{(\beta^{+}-\beta^{-})} \quad (attribues \quad cardinales) \\ i,j \in \{1,...,N\}\,; l \in \{1,...,S\} \end{cases} \tag{4}$$

$\alpha^{+} = \max\limits_{1\leq i,j\leq N}\left(\alpha_{ij}^{l}\right)$; $\beta^{+} = \max\limits_{1\leq i,j\leq N}\left(\beta_{ij}^{l}\right)$, $\alpha^{-} = \min\limits_{1\leq i,j\leq N}\left(\alpha_{ij}^{l}\right)$ et $\beta^{-} = \min\limits_{1\leq i,j\leq N}\left(\beta_{ij}^{l}\right)$ et $l \in \{1,...,S\}$.

Step 5: Normalised overall dominance index $delta_{ij}$ and d_{ij} of actions

$$\begin{cases} \alpha_{ij} = \sum\limits_{l=1}^{S}\left[\delta_{ij}^{[l]} - \delta_{ij}^{[l-1]}\right] \times \mu(\{l\}) + \mu(\{l+1\}) + ... + \mu(\{S\}) \\ d_{ij} = \max\limits_{1\leq k\leq K}\left[\min(d_{ij}^{[k]}; \mu(\{k\},\{k+1\},...,\{K\}))\right] \\ i,j = 1,\ldots,N, l = 1,\ldots,,S, \end{cases} \tag{5}$$

where [.] is a permutation such that $\delta_{ij}^{[1]} < \delta_{ij}^{[2]} < ... < \delta_{ij}^{[K]}$ et $d_{ij}^{[1]} < d_{ij}^{[2]} < ... < d_{ij}^{[K]}$

Step 6 Calculate the total dominance of action i over action j.

$$\begin{cases} D_{ij} = P_O \times \alpha_{ij} + P_C \times d_{ij} \qquad i,j \in \{1,...,M\} \\ P_O = \sum\limits_{j\in O} P_j \quad et \quad P_C = \sum\limits_{j\in C} P_j \end{cases} \tag{6}$$

Step 7 Calculating the overall score by action.

$$S_i = \left[\sum_j \frac{D_{ji}}{D_{ij}}\right]^{-1} \qquad i,j \in \{1,...,M\} \tag{7}$$

$S(a_i) > S(a_j)$ means that share a_i is better than share a_j.

3.1 Application to the choice of the best site

We obtain a family of criteria: $C = \{c_1, c_2, c_3, c_4\}$ where c_1 is the reception capacity of the site, c_2 the cost of developing the site, c_3 the resistance of the site to climatic hazards, c_4 the duration of the work and a family of alternatives: A={pabre site, famagan site, nioko site, loumbila site }. Each of these three decision-makers constructs its judgement matrix as follows:

Table 1: decision maker 1(the coordinator)/D1

	c_1	c_2	c_3	c_4
weight	6	5	4	7
pabre	good	5	not very important	7
famagan	quite good	6	important	7
nioko	fair	7	very important	1
loumbila	fair	4	less important	2

Table 2: decision-maker 2(the engineer)/D2

	c_1	c_2	c_3	c_4
weight	4	3	7	5
pabre	quite good	2	very important	4
famagan	fair	7	very important	3
nioko	good	8	not very important	1
loumbila	fair	1	not very important	2

Table 3: decision maker 3(the director)/D3

	c_1	c_2	c_3	c_4
weight	1	5	2	4
pabre	good	2	not very important	6
famagan	very good	8	less important	3
nioko	fair	4	very important	7
loumbila	good	3	less important	4

Table 4: Number of years of experience of decision-makers

	d_1	d_2	d_3
Year of experience	5	8	3

Table 5: Level of importance of each decision-maker

	$\{d_1\}$	$\{d_2\}$	$\{d_3\}$	$\{d_1, d_2\}$	$\{d_1, d_3\}$	$\{d_2, d_3\}$	$\{d_1, d_2, d_3\}$
measure μ	0.33	0.46	0.2	0.79	0.53	0.66	1

Table 6: **matrix of overall criteria weights**

	d_1	d_2	d_3	$P_j = \left(\frac{\sum_{k=1}^{3} (p_j^k)^{1/3}}{n} \right)^3$
c_1	6	4	1	3.16
c_2	5	3	5	4.25
c_3	4	7	2	3.99
c_4	7	5	4	5.23

Table 7: **Index of dominance of actions in the cardinal criteria of D1**

	pabre	famagan	nioko	loumbila
pabre	0	-5	32	40
famagan	5	0	37	45
nioko	-32	-37	0	8
loumbila	-40	-45	-8	0

Table 8: **Index of dominance of actions in the D1 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0	2	2	2
famagan	-2	0	2	10
nioko	-2	-2	0	4
loumbila	-2	-10	-4	0

Table 9: **Action dominance index in D2 cardinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0	-10	-3	13
famagan	10	0	7	23
nioko	3	-7	0	16
loumbila	-13	-23	-16	0

Table 10: **Action dominance index in D2 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0	4	3	7
famagan	-4	0	3	3
nioko	-3	-3	0	4
loumbila	-7	-3	-4	0

Table 11: **Index of dominance of actions in the cardinal criteria of D3**

	pabre	famagan	nioko	loumbila
pabre	0	-18	-14	3
famagan	18	0	4	21
nioko	14	-4	0	17
loumbila	-3	-21	-17	0

Table 12: **Index of dominance of actions in the D3 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0	-3	-1	-2
famagan	3	0	-1	1
nioko	1	1	0	2
loumbila	2	-1	-2	0

Table 13: **Index of differential dominance of actions in the D1 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0,5	0.6	0.6	0.6
famagan	0.4	0.5	0.6	1
nioko	0.4	0.4	0.5	0.7
loumbila	0.4	0	0.3	0.5

Table 14: **Index of differential dominance of actions in the D2 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0,5	0.78	0.71	1
famagan	0.21	0.5	0.71	0.71
nioko	0.28	0.28	0.5	0.78
loumbila	0	0.28	0.21	0.5

Table 15: **Index of differential dominance of actions in the D3 ordinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0,5	0	0.33	0.16
famagan	1	0.5	0.33	0.66
nioko	0.66	0.66	0.5	0.83
loumbila	0.83	0.33	0.16	0.5

Table 16: **Index of differential dominance of actions in the D1 cardinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0.5	0.44	0.85	0.94
famagan	0.55	0.5	0.91	1
nioko	0.14	0 .08	0.5	0.58
loumbila	0.05	0	0.41	0.5

Table 17: **Index of differential dominance of actions in the D2 cardinal criteria**

	pabre	famagan	nioko	loumbila
pabre	0.5	0.28	0.43	0.78
famagan	0.71	0.5	0.65	1
nioko	0.56	0.34	0.5	0.84
loumbila	0.21	0	0.15	0.5

Table 18: **Index of differential dominance of actions in the cardinal criteria of D3**

	pabre	famagan	nioko	loumbila
pabre	0.5	0.07	0.16	0.57
famagan	0.92	0.5	0.59	1
nioko	0.83	0.40	0.5	0.90
loumbila	0.42	0	0.09	0.5

Table 19: **Index of overall differential dominance of stocks in cardinal criteria**

	α_{11}	α_{12}	α_{13}	α_{14}	α_{21}	α_{22}	α_{23}	α_{24}
d_1	0.5	0.44	0.85	0.94	0.55	0.5	0.91	1
d_2	0.5	0.28	0.43	0.78	0.71	0.5	0.65	1
d_3	0.5	0.07	0.16	0.57	0.92	0.5	0.59	1
$\alpha_{ij} = \sum_{l=1}^{3} \left[\delta_{ij}^{[l]} - \delta_{ij}^{[l-1]}\right] \times [\mu(\{l\}) + \mu(\{l+1\}) + ... + \mu(\{3\})]$	0.5	0.24	0.42	0.74	0.70	0.5	0.68	1

	α_{31}	α_{32}	α_{33}	α_{34}	α_{41}	α_{42}	α_{43}	α_{44}
d_1	0.14	0.08	0.5	0.58	0.05	0	0.41	0.5
d_2	0.56	0.34	0.5	0.84	0.21	0	0.15	0.5
d_3	0.83	0.40	0.5	0.90	0.42	0	0.09	0.5
$\alpha_{ij} = \sum_{l=1}^{3} \left[\delta_{ij}^{[l]} - \delta_{ij}^{[l-1]}\right] \times [\mu(\{l\}) + \mu(\{l+1\}) + ... + \mu(\{3\})]$	0.47	0.27	0.5	0.59	0.2	0	0.18	0.5

Table 20: **Index of overall differential dominance of actions in ordinal criteria**

	α_{11}	α_{12}	α_{13}	α_{14}	α_{21}	α_{22}	α_{23}	α_{24}	α_{31}	α_{32}
d_1	0.5	0.6	0.6	0.6	0.4	0.5	0.6	1	0.4	0.4
d_2	0.5	0.78	0.71	1	0.21	0.5	0.71	0.71	0.28	0.28
d_3	0.5	0.78	0.71	1	0.21	0.5	0.71	0.71	0.28	0.28
$d_{ij} = \max_{1 \leq k \leq 3} \left[\min(d_{ij}^{[k]}; \mu(\{k\}, \{k+1\}, ..., \{3\}))\right]$	0.5	0.66	0.66	0.66	0.21	0.5	0.66	0.71	0.28	0.28

	α_{33}	α_{34}	α_{41}	α_{42}	α_{43}	α_{44}
d_1	0.5	0.7	0.4	0	0.3	0.5
d_2	0.5	0.78	0	0.28	0.21	0.5
d_3	0.5	0.78	0	0.28	0.21	0.5
$d_{ij} = \max_{1 \leq k \leq 3} \left[\min(d_{ij}^{[k]}; \mu(\{k\}, \{k+1\}, ..., \{3\}))\right]$	0.5	0.7	0.2	0.28	0.21	0.5

Table 21: **Total dominance**

D11	D12	D13	D14	D21	D22	D23	D24	D31	D32	D33	D34	D41	D42	D43	D44
8.32	7.09	8.85	11.84	8.23	8.32	11.27	14.61	6.59	4.63	8.32	10.66	3.38	2.04	3.29	8.32

Table 22: textbfThe overall score and ranking of actions

S1(pabre)	S2(famagan)	S3(nioko)	S3(loumbila)
0.31	0.22	0.19	0.06
1^{st}	2^{nd}	3^{rd}	4^{th}

4 Results

Using the extension of the EVAMIX method to solve multi-decider multicriteria problems with an order of importance for each decision-maker produces a ranking of alternatives from best to worst. It also reduces the compensation between strong and weak criteria.

5 Limit

Like any aggregation function, our method certainly has its limits. An implementation could give us a better idea of these limits. In addition, the lack of a comparison with other existing methods in the literature could constitute a limitation.

6 Conclusion

The EMEBICS method was used to solve a group decision problem using the order of importance of the decision-makers. The simulation and application enabled us to identify the Pabre site as the best choice. However, like all methods, it is not without its shortcomings. Our future research will therefore focus on its implementation, algorithmic complexity and the robustness of the results obtained.

Bibliography

[1] Abbas, M. and Chergui, Z.(2017). Performance of multicriteria decision making methods: Study and cases, Int.J. Multicriteria Decision Making, Vol.7, No. 2,pp.116-145.

[2] Veera P. Darji, Ravipudi V. Rao *Application of AHP/EVAMIX Method for Decision Making in the Industrial Environment, Department of Mechanical Engineering, S.V. National Institute of Technology, Surat, India Email: veera.jani@rediffmail.com, ravipudirao@gmail.com, American Journal of Operations Research, 2013, 3, 542-569 http://dx.doi.org/10.4236/ajor.2013.36053*

[3] N. Bakhta. Multi-agent model for the design of collective decision support systems. PhD thesis, University of Oran, 2013-2014.

[4] Jean-Luc Marichal. Aggregation functions for decision making. Applied Mathematics Department, University of Luxembourg 162A, avenue of the Faïencerie, L-1511 Luxembourg, 2005, Email: jean-luc.marichal[at]uni.lu

[5] BELARIBI Fatima Zohra(2021). *Use of multi-criteria decision-support methods in water resource management: Application of the PROMETHEE method in the selection of a seawater desalination process,thesis, Abou Bekr Belkaid University*