FULL RANK VOTING: THE CLOSEST TO VOTING WITH INTENSITY OF PREFERENCES

Luis G. Vargas

356 Mervis Hall
University of Pittsburgh, Pittsburgh, PA, USA
lgvargas@pitt.edu

Marcel C. Minutolo

Robert Morris University, Moon, PA, USA
minutolo@rmu.edu

Highlights

- This research shows how intensity of preference can be captured in the voting process provided that full rank voting is used.
- We show through simulation that voting with intensity of preferences from Saaty's 1-9 scale is equivalent to full rank voting.
- Voting is not group decision making. Group decision making attempts to reach consensus.

ABSTRACT

Voting is a fundamental aspect of democracy, but traditional voting schemes often fail to capture the intensity of preferences individuals possess. This loss of intensity can lead to an oversimplification of complex issues and a lack of accurate representation of diverse opinions. To address this limitation, we propose a voting method called "Full Rank Voting" that incorporates the intensity of preferences into the voting process. By using pairwise comparisons and Saaty's funda-mental scale, we transform individual preferences into numerical values and construct a matrix that represents the intensity of preferences for different candidates or options. In the case of two candidates, each voter expresses their preference intensity by assigning a numerical value from the fundamental scale. These values are then used to calculate the priorities or percentages of votes for each candidate. By incorporating intensity of preferences, the voting process becomes more nuanced, and ordinal preferences become a specific case of cardinal preferences. When multiple candidates are involved, we encounter the challenge of combining intensity of preferences with rank voting. We conduct simulation experiments to demonstrate that rank voting and voting with intensity of preferences yield similar results, even for relatively small sample sizes. We then apply the process to an actual voting dataset to further demonstrate the results. Overall, Full Rank Voting offers a solution for capturing the intensity of preferences in voting, leading to a more accurate representation of individual choices, increased democratic legitimacy, and the ability to identify common ground and prioritize preferences based on their strength.

Keywords (3-6): Voting; Rank; Comparisons; Preferences; Intensity

1. Introduction

Voting is the basis of democracy; one person one vote. However, not everybody likes (dislikes) a candidate with the same intensity though this is lost in most voting schemes. It is important for a democracy to capture the intensity of preferences since knowing it provides a more accurate representation of choice and more nuanced understanding of individual preferences. There are many reasons why it is important for a democracy to capture intensity of preferences when voting. For instance, it reflects diversity of opinion. By capturing intensity, we can recognize the varying degrees of support or opposition individuals may have toward different options. This ensures that the diversity of opinions within a population are accurately represented.

Voting is not the same as group decision making that may require achieving a consensus. To achieve a consensus, if the intensity of preferences is represented using a numerical scale, the numerical preferences may be required to be close to each other so that the synthesis represents the group's preference. Voting does not require synthesis of numerical preferences, just account for the vote in whatever form it is provided.

When we capture the intensity of preferences, we enhance democratic legitimacy. Since a democracy seeks to reflect the will of the people, when intensity of preferences is considered, we get a more accurate measurement of collective will. Knowing more accurately the collective will helps to increase the legitimacy of the outcomes, as they align more closely with the true sentiments of the electorate. Treating all preferences equally without considering their intensity can lead to oversimplification of complex issues; it fails to account for the strength of convictions individuals have towards specific choices. By capturing intensity, we gain a deeper understanding of the underlying motivations and beliefs behind people's preferences.

2. Literature Review

To represent intensity of preferences we need to be able to measure our preferences in a scale. We usually use words to represent how strongly we prefer something, for instance, I like it a lot, very much, not all, and so on. These words need to be associated with numbers so that we can combine the preferences to represent how something is liked or preferred. We could count how many people in a group prefer something strongly, but counting does not represent how strongly two individuals prefer the item or the candidate. To be able to represent intensity numerically we need to use measurement. One could say a person prefers A to B strongly, and another could say that she prefers A to B equally. We cannot mathematically combine the intensities "strongly" and "equally" because they belong to a nominal scale. However, if they were to be assigned a numerical value that satisfy some ordinal condition, like the number assigned to "strongly" must be greater than the number assigned to "equally," we could try to transform the entire set of preferences of a group into a numerical value. Clearly, those numerical assignments may not be the same for everybody in a group. Because that scale may not be the same for everyone, we may never agree on the definition of the unit of measurement. Thus, we need relative measurement which does not use units; a simple example can explain this. There is another school of thought which models preferences by linguistic preference relations with fuzzy sets, e.g., (Herrera-Viedma et al., 2005; Herrera et al., 2000; Zadeh, 1975a; Zadeh, 1975b; Zadeh, 1975c; Zhou et al., 2008).

Consider a set of stones that we need to rank in terms of weight (Saaty & Vargas, 2007), but we do not have a gadget to ascertain their weight. We can take the stones in our hands and guess which one is heavier, but to do that we need to select them in pairs and order them in terms of the perceived weight. However, ordering them does not allow us to find their relative weights. To find the relative weights, we need to assign to each paired comparison a numerical value that somehow reflects our perception of weights such as one stone is heavier than the other and, in this case, we think that one is 3 times heavier than the other. This would mean that the total weight of three stones the size of the smaller stone would be equal to the weight of the larger stone. Thus, we need to build a scale that maps the words representing intensity to numbers. This subject falls under the disciple known as psychophysics (Fechner, 1966). Gustav Theodor Fechner, a German philosopher and scientist, developed the area of psychophysics. Psychophysics investigates the quantitative associations between mental and physical phenomena, focusing on the precise correlation between sensory perceptions and the external stimuli generating them. Saaty (1980) developed his fundamental 1-9 scale using psychophysics principles. In what follows, we express numerical preferences by Saaty's 1-9 scale.

3. Hypotheses/Objectives

In this paper we put forth a voting method, we call it Full Rank Voting, that captures intensity of preferences in the democratic process when intensity is expressed by Saaty's fundamental scale. Voting is not the same as decision making by group consensus. Voting needs to capture the preferences of everyone in a group. Traditional voting systems often fail to capture the varying degrees of support or opposition individuals have towards different options, leading to an oversimplification of complex issues and a lack of representation of diverse opinions. By incorporating the intensity of preferences into the voting process, we can enhance democratic legitimacy, improve the accuracy of collective will, and facilitate compromise and consensus-building.

4. Research Design/Methodology

We conducted a simulation experiment to verify the convergence of pairwise voting matrices as the sample size (number of voters) increases. The results showed that as the sample size grows, the matrices obtained from rank voting and intensity of preferences approach close values. This suggests that rank voting provides reliable results that are comparable to the outcomes obtained from the actual count of votes, even for relatively small sample sizes.

The properties of Full Rank Voting are the properties of the Principal Right Eigenvector Method which were more fully developed in (Vargas, 2016) but herein we provide a summary of the approach. We are not able to infer that alternative *i* beats alternative *j* from $a_{ij}(\phi) \equiv \dfrac{v_{ij}(\phi)}{v_{ji}(\phi)} > 1$ if the voting matrix of pairwise comparison ratios does not satisfy *row dominance*. A reciprocal pairwise voting matrix $A(\phi) = \left\{a_{ij}(\phi)\right\}$ satisfies *row dominance* if for any two rows *i* and *j*, $a_{ih}(\phi) \geq a_{jh}(\phi)$ or $a_{ih}(\phi) \leq a_{jh}(\phi)$, for all *h*. Hence, a *profile is row dominant* when the corresponding reciprocal pairwise voting matrix is row dominant. Additionally, $v_{ih}(\phi) + v_{hi}(\phi) = v_{jh}(\phi) + v_{hj}(\phi) = N$, $a_{ih}(\phi) \geq a_{jh}(\phi)$ implies $v_{ih}(\phi) \geq v_{jh}(\phi)$. Row dominance defines a strong order on the set of alternatives.

1. The PR-eigenvector method applied to profiles satisfying row dominance identifies the Condorcet winner.
2. The PR-eigenvector method applied to profiles that satisfy row dominance iss consistent. A voting method *f* is *consistent* when given two disjoint profiles, ϕ' and ϕ'', it yields the same consensus ordering, $f(\phi') = f(\phi'')$, resulting in the same consensus ordering on the joint profile $\phi = \phi' \cup \phi''$, $f(\phi) = f(\phi') = f(\phi'')$. The combination of two separate profiles that satisfy row dominance and produce identical orderings for alternatives also satisfies row dominance and results in the same ordering.
3. The PR-Eigenvector method on profiles with row dominance satisfies independence from irrelevant alternatives. A voting method adheres to the principle of independence from irrelevant alternatives (IIA) when the inclusion or removal of an alternative in a profile does not modify the consensus ordering obtained from the original profile.
4. Further, the PR-Eigenvector method on profiles that satisfy row dominance satisfies the independence of *clones* criterion. A clone refers to a candidate that is identical to another in the pool implying that it neither dominates nor is dominated by the other alternatives. Clones do not alter the preferences among alternatives, ensuring that the resulting reciprocal pairwise voting matrix continues to satisfy row dominance. This condition leads to the expected result.

It is worthwhile to point out that because the PR-eigenvector method satisfies all the properties included in Arrow's impossibility theorem (Saaty & Vargas, 2012), Full Rank Voting is not a counterexample to Arrow's theorem.

5. Results/Model Analysis

In 2018 the state of Main in the USA used Rank Choice voting in the primary elections. One of them was for Governor of the state. The following table (Table 1) taken from the website[1] shows the results of the primary election for governor of the state. We were able to obtain the rank choice raw data from the same website.

Table 1. Results of 2018 Democratic Primary Election

Election Name	**Democratic Primary Election**											
Election Date	**June 12, 2018**											
Office Title	**Governor**											
	Round 1			**Round 2**			**Round 3**			**Round 4**		
Candidate Names	**Votes**	**Percentage**	**Transfer**	**Votes**	**Percentage**	**Transfer**	**Votes**	**Percentage**	**Transfer**	**Votes**	**Percentage**	**Transfer**
Cote, Adam Roland	35478	28.13%	**2065**	37543	30.25%	**5080**	42623	34.79%	**11243**	53866	45.94%	0
Dion, Donna J.	1596	01.27%	-1596	0	00.00%	0	0	00.00%	0	0	00.00%	0
Dion, Mark N.	5200	04.12%	-5200	0	00.00%	0	0	00.00%	0	0	00.00%	0
Eves, Mark W.	17887	14.18%	**1634**	19521	15.73%	-19521	0	00.00%	0	0	00.00%	0
Mills, Janet T.	**41735**	**33.09%**	**2307**	**44042**	**35.49%**	**5903**	**49945**	**40.77%**	**13439**	**63384**	**54.06%**	**0**
Russell, Diane Marie	2728	02.16%	-2728	0	00.00%	0	0	00.00%	0	0	00.00%	0
Sweet, Elizabeth A.	20767	16.46%	**2220**	22987	18.52%	**6957**	29944	24.44%	-29944	0	00.00%	0
Write-in	748	00.59%	-748	0	00.00%	0	0	00.00%	0	0	00.00%	0
Ballot Exhausted												
By Overvotes	430		42	472		35	507		73	580		0
By Undervotes	5681		1887	7568		1488	9056		5099	14155		0
By Exhausted Choices	0		117	117		58	175		90	265		0
Continuing Ballots	126139		0	124093		0	122512		0	117250		0
TOTAL	**132250**		**0**	**132250**		**0**	**132250**		**0**	**132250**		**0**
Winning threshold by round	63070			62047			61257			58626		
Generated: 06/21/2018 18:45												

The data contains 132,138 votes. For a vote to be acceptable the voter did not have to rank all the candidates, just one. We discarded all votes that did not have a choice, and we ended up with a sample of 126,147 votes with at least one choice. The counts in the table above represent the number of votes that rank the candidate first. For example, 35,478 voters selected Mr. Adam R. Cote as their first choice. Using rank choice voting, the candidates with less votes were systematically eliminated, and their votes distributed to the remaining candidates based on how the votes eliminated rank the other candidates they did not rank first. After the fourth round two candidates remained. One of the candidates, Ms. J.T. Mills received more than fifty percent of the votes and was declared the winner of the primary election.

We use these data to show that intensity of preference in voting is only captured when Full Rank voting is used, i.e., a vote is acceptable if and only if all the candidates are ranked. This is the method employed in Australian parliamentary elections[2]. They refer to it as preferential voting.

---

[1] [Bureau of Corporations, Elections & Commissions, Elections and Voting, Results (maine.gov)]()

[2] https://www.aec.gov.au/

Thus, if ties are not allowed the pairwise voting matrix is not close to the matrix generated with intensity of preferences (See Tables 2 and 3)

Table 2. Priorities from Pairwise Voting Matrix w/o Ties

	A	B	C	D	E	F	G	PR-Eigenvector	Winning Candidates
A	1.0000	5.2405	3.0316	1.2901	0.8412	3.9542	1.2313	0.2138	44.42%
B	0.1908	1.0000	0.5013	0.2255	0.1429	0.7908	0.1760	0.0375	
C	0.3299	1.9947	1.0000	0.4064	0.2679	1.6240	0.4029	0.0732	
D	0.7751	4.4339	2.4609	1.0000	0.6465	3.9523	1.0162	0.1763	
E	1.1888	6.9957	3.7327	1.5468	1.0000	5.2804	1.6008	0.2674	55.58%
F	0.2529	1.2645	0.6158	0.2530	0.1894	1.0000	0.2374	0.0475	
G	0.8122	5.6823	2.4820	0.9841	0.6247	4.2131	1.0000	0.1844	

Table 3. Priorities from Matrix of Intensity of Preferences w/o Ties

	A	B	C	D	E	F	G	PR-Eigenvector	Winning Candidates
A	1.0000	1.8141	1.5046	0.9432	0.7443	1.6903	0.9457	0.1393	35.68%
B	0.5512	1.0000	0.2275	0.1554	0.1150	0.2756	0.1223	0.0315	
C	0.6646	4.3965	1.0000	0.2827	0.2199	0.5626	0.2879	0.0709	
D	1.0602	6.4345	3.5377	1.0000	0.5513	1.4320	0.7188	0.1632	
E	1.3435	8.6978	4.5475	1.8141	1.0000	2.3813	1.2713	0.2511	64.32%
F	0.5916	3.6284	1.7776	0.6983	0.4199	1.0000	0.1629	0.0906	
G	1.0574	8.1785	3.4733	1.3913	0.7866	6.1390	1.0000	0.2534	

On the other hand, when ties are allowed the matrix of counts and the matrix of intensity of preferences are very close (See Tables 4 and 4).

Table 4. Priorities from Pairwise Voting Matrix w Ties Allowed

	A	B	C	D	E	F	G	PR-Eigenvector	Winning Candidates
A	1.0000	3.1828	2.3470	1.2452	0.8507	2.7694	1.2010	0.2003	44.56%
B	0.3142	1.0000	0.6891	0.3678	0.2360	0.8918	0.3246	0.0596	
C	0.4261	1.4511	1.0000	0.5123	0.3401	1.3212	0.5023	0.0860	
D	0.8031	2.7188	1.9520	1.0000	0.6745	2.5704	1.0134	0.1672	
E	1.1755	4.2366	2.9402	1.4827	1.0000	3.6620	1.5318	0.2491	55.44%
F	0.3611	1.1213	0.7569	0.3890	0.2731	1.0000	0.3798	0.0670	
G	0.8327	3.0804	1.9909	0.9868	0.6528	2.6330	1.0000	0.1708	

Table 5. Priorities from Matrix of Intensity of Preferences w Ties Allowed

	A	B	C	D	E	F	G	PR-Eigenvector	Winning Candidates
A	1.0000	3.1848	2.3453	1.2439	0.8504	2.7707	1.2021	0.2003	44.58%
B	0.3140	1.0000	0.6894	0.3675	0.2363	0.8920	0.3248	0.0596	
C	0.4264	1.4505	1.0000	0.5113	0.3404	1.3229	0.5025	0.0860	
D	0.8039	2.7208	1.9558	1.0000	0.6745	2.5677	1.0124	0.1673	
E	1.1760	4.2322	2.9375	1.4825	1.0000	3.6590	1.5272	0.2489	55.42%
F	0.3609	1.1211	0.7559	0.3895	0.2733	1.0000	0.3798	0.0670	
G	0.8319	3.0790	1.9899	0.9878	0.6548	2.6328	1.0000	0.1709	

6. Conclusions

Voting is not the same as decision making by group consensus. Voting needs to capture the preferences of everyone in a group. Traditional voting systems often fail to capture the varying degrees of support or opposition individuals have towards different options, leading to an oversimplification of complex issues and a lack of representation of diverse opinions.

We showed that using Full Rank Voting the voters can rest assured that they are expressing how strongly they prefer candidates, not just in general, but under a variety of criteria. The method presented here can be easily extended to evaluate real political candidacies.

7. Limitations

A possible limitation could be if the number of voters is small. However, we have used the method in a group as small as 32 people and the conclusions appear to be stable. More experiments need to be conducted to affirm that the method holds in small groups if one where to use it in a corporate environment.

8. Key References

Fechner, G. (1966). *Elements of Psychophysics* (Vol. 1). Holt, Rinehart and Winston.

Herrera-Viedma, E., Martinez, L., Mata, F., & Chiclana, F. (2005). A consensus support system model for group decision-making problems with multigranular linguistic preference relations. *IEEE Trans. Fuzzy Systems, 13*, 644-658.

Herrera, F., Herrera-Viedma, E., & Chiclana, F. (2000). Linguistic decision analysis: steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems, 115*, 67-82.

Saaty, T. L., & Vargas, L. G. (2007). Dispersion of group judgments. *Mathematical and Computer Modelling, 46*, 918-925.

Saaty, T. L., & Vargas, L. G. (2012). The Possibility Of Group Choice: Pairwise Comparisons and Merging Functions. *Social Choice and Welfare, 38*(3), 481-496.

Vargas, L. G. (2016). Voting with Intensity of Preferences. *International Journal of Information Technology & Decision Making, 15*(4), 839-859.

Zadeh, L. A. (1975a). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences, 8*, 199-249.

Zadeh, L. A. (1975b). The concept of a linguistic variable and its application to approximate reasoning-II. *Information Sciences, 8*, 301-357.

Zadeh, L. A. (1975c). The concept of a linguistic variable and its application to approximate reasoning-III. *Information Sciences, 9*, 43-80.

Zhou, S.-M., Chiclana, F., John, R. I., & Garibaldi, J. M. (2008). Type-1 OWA operators for aggregating uncertain information with uncertain weights induced by type-2 linguistic quantifiers. *Fuzzy Sets and Systems, 159*, 3281-3296.