Extension of the Simple Multi-Attribute Rating Technique (SMART) to group decision problem solving

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Abstract

This study proposes an extension of the SMART (Simple Multi-Attribute Rating Technique) method to adapt it to group decisions, a context where classical methods often show their limits. To better manage the diversity of preferences and complex interactions between group members, it integrates the WOWA (Weighted Ordered Weighted Average) aggregation operators and the exponential weighted logarithmic mean. The use of WOWA allows performances to be weighted according to their relative importance and order, while the exponential logarithmic mean helps to effectively manage extreme values. This adaptation of SMART aims to foster harmonious collective decision-making, and offers organizations a flexible tool for collaborative governance processes, particularly useful in environments marked by complex dynamics and diverse viewpoints.

Keywords : group decision-making, SMART extension, aggregation

1 Introduction

Collective decision-making plays a crucial role in the functioning of human societies and multi-agent systems[3]. The Simple Multi-Attribute Rating Technique (SMART) is a decision evaluation technique that takes several attributes into account when evaluating alternatives [4, 6]. It has established itself as a reliable tool for evaluating individual decisions, thanks to its simplicity[1].

In this study, we focus on the limitations of conventional decision-making methods, particularly in group decision-making contexts where the diversity of preferences and complex interactions make the aggregation of opinions more delicate. This work was motivated by the need to adapt the SMART method, commonly used to evaluate alternatives in individual contexts, to collective decisions, an area where it shows limitations.

The general aim of this study is therefore to propose an extension of the SMART method to better meet the challenges of collective decision-making, by integrating advanced aggregation operators such as WOWA (Weighted Ordered Weighted Average) and the exponential weighted logarithmic mean. The choice of these operators makes it possible to effectively manage the diversity of opinions, by flexibly weighting performances and limiting the impact of extreme values. This research is important because it offers organizations an improved tool for collaborative governance, fostering harmonious decisions that are better adapted to complex group dynamics.

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2 Literature Review

2.1 Aggregation operators

2.1.1 Weighted Ordered Weighted Average (WOWA)

The WOWA aggregation operator was introduced by Torra [8], who uses a weighting vector $p \in \mathbb{R}^n$ for the criteria.
This aggregator is defined by the relation (2.1) :

$$\begin{aligned} \mathrm{WOWA}(x) &= \sum_{i=1}^{n} \left[x_{\sigma(i)} - x_{\sigma(i-1)}\right] \varphi\left(\sum_{k=i}^{n} p_{\sigma(k)}\right) \qquad \text{with } x \in \mathbb{R}^n \\ &= \sum_{i=1}^{n} \left[\varphi\left(\sum_{k=i}^{n} p_{\sigma(k)}\right) - \varphi\left(\sum_{k=i+1}^{n} p_{\sigma(k)}\right)\right] x_{\sigma(i)} \end{aligned} \tag{2.1}$$

where σ is the permutation that reorders the components of x in ascending order, i.e. $x_{\sigma(1)} \leq x_{\sigma(2)} \leq \cdots \leq x_{\sigma(n)}$ and $x_{\sigma(0)} = 0$; the function φ is strictly increasing and such that $\varphi(0) = 0$.

2.1.2 Exponential logarithmic mean B_k

The exponential logarithmic mean operator [2] is a commonly used aggregation operator in information theory. It is defined by the following relation (2.2) :

$$B_k\left(x_1, \ldots, x_n\right) = k \log\left(\frac{\sum_{i=1}^{n} \exp\left(\frac{x_i}{k}\right)}{n}\right) \tag{2.2}$$

- If $k \to 0$, we recover the arithmetic mean : $\lim\limits_{k \to 0} B_k\left(x_1, \ldots, x_n\right) = \frac{1}{n} \sum\limits_{i=1}^{n} x_i$
- If $k \to +\infty$, we obtain : $\lim\limits_{k \to +\infty} B_k\left(x_1, \ldots, x_n\right) = \max\left(x_1, \ldots, x_n\right)$

2.2 Description of the SMART method

The Simple Multi-Attribute Rating Technique (SMART) is a multi-criteria evaluation technique that was developed in 1977 by Edward and helps in decision-making by evaluating different options across multiple criteria [7, 4].
Here are the general steps of the SMART method :

1. **Assigning weights to the criteria and normalization**
2. **Evaluation of the alternatives**
   The utility values are calculated based on the following formulas :

   - $$u_j(a_i) = \frac{g_{ij} - \min\limits_j \{g_{ij}\}}{\max\limits_j \{g_{ij}\} - \min\limits_j \{g_{ij}\}} \qquad \textbf{beneficial (criterion to maximize)} \tag{2.3}$$
   - $$u_j(a_i) = \frac{\max\limits_j \{g_{ij}\} - g_{ij}}{\max\limits_j \{g_{ij}\} - \min\limits_j \{g_{ij}\}} \qquad \textbf{non-beneficial (criterion to minimize)} \tag{2.4}$$

   where : $u_j(a_i)$ is the utility value of alternative i with respect to criterion j ;
   g_{ij} is the value of alternative i with respect to criterion j.

3. **Calculation and aggregation of weighted scores**
   In this step, the weighted scores for each alternative are calculated and then aggregated.

$$U(a_i) = \sum_{j=1}^{m} \widehat{w}_j u_j(a_i) \qquad \text{with} \quad i = 1, 2, \cdots, n. \tag{2.5}$$

   where $U(a_i)$ is the total utility value of alternative i and n is the number of alternatives.
4. **Selection of the best alternative**
   The best alternative is obtained using the relation (2.6). This alternative is considered the most favorable based on the criteria and their respective weights.

$$\max_{\forall i} \sum_{j=1}^{m} \widehat{w}_j u_j(a_i) \qquad \text{or} \qquad \max_{\forall i} \{U(a_i)\} \tag{2.6}$$

   We then have the following preference : $a_i \succsim a_k \Longleftrightarrow U(a_i) \geq U(a_k)$

3 Hypotheses/Objectives

The objective of this research is to adapt the SMART method to group decision-making by incorporating the WOWA operators and the weighted exponential logarithmic mean. This extension aims to improve the quality of collective decisions by considering the order of preferences and, in particular, the impact of extreme values. This approach will lead to more harmonious and representative decisions, promoting cohesion and satisfaction within the group in collaborative governance contexts.

4 Research Design/Methodology

4.1 Formulation of the problem

Consider the decision support problem involving multiple criteria and multiple decision makers below. Such a problem occurs when you have the following five sets : :
▷ $D = \{d_1, d_2, \ldots, d_s\}$ with $s \geq 2$: set of all s decision makers ;
▷ $A = \{a_1, a_2, \ldots, a_n\}$ with $n \geq 2$: denotes a collection of n alternatives or actions ;
▷ $C = \{c_1, c_2, \ldots, c_m\}$ with $m \geq 2$: designating the m criteria selected ;
▷ $X = \left\{g_{ij}^k, i = 1, \ldots, n, j = 1, \ldots, m, k = 1, \ldots, s\right\}$ designating the performance of the alternative i on the criterion j for the k^{th} decision maker.

4.2 Principle of MAC-SMART

The principle of MAC-SMART (Collective Aggregation Method based on the SMART) relies on the use of two specific operators to aggregate the information from group members in order to obtain a global preference matrix. MAC-SMART is based on the aggregation of criterion weights through the weighted exponential logarithmic mean denoted as B_k, and the aggregation of the performance of alternatives via the WOWA operator, in order to reflect collective preferences.

4.3 Presentation of the new method : MAC-SMART

Step 1 : **Determination of the global weights of the criteria**
In this section, the global weight w_j of each criterion c_j is first calculated using the operator B_k given by equation (2.2).

$$w_j = B_k\left(w_j^1, w_j^2, \ldots, w_j^s\right) \tag{4.1}$$

where w_j^k is the weight of criterion j assigned by decision-maker k.
These global weights w_j form the matrix of global weights w, which is presented as follows :

$$w = \begin{pmatrix} w_1 & w_2 & \ldots & w_m \end{pmatrix} \tag{4.2}$$

Step 2 : **Determination of the global evaluations of the alternatives**
This section determines the global evaluation g_{ij} of each alternative a_i according to the criterion c_j, based on the WOWA operator given by relation (2.1).
To calculate the global performance g_{ij} of alternative i with respect to criterion j, we proceed as follows :

$$g_{ij} = \text{WOWA}\left(g_{ij}^1, g_{ij}^2, \ldots, g_{ij}^s\right)$$

This gives us the matrix of global performances D as follows :

$$D = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} \end{pmatrix}$$

Step 3 : Formation of the global preference matrix
The global preference matrix, denoted D_{glob}, is formed from the matrices w and D.

$$D_{glob} = \begin{pmatrix} w \\ D \end{pmatrix} \qquad \Longrightarrow \qquad D_{glob} = \begin{pmatrix} w_1 & w_2 & \cdots & w_m \\ g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} \end{pmatrix}$$

Step 4 : Application of SMART to the global preference matrix
In this step, the SMART method described earlier is simply applied to the global decision matrix D_{glob}.

5 Results/Model Analysis

This digital experiment is taken from [5].

5.1 Problem statement

The problem is to determine the best care center for severe cases of covid-19. For this, we have :
- Four centers : Center 1, Center 2, Center 3, Center 4 ;
- Five criteria : Respirator equipment (Equi.Resp), Bed equipment (Equi.Lit), Qualification of personnel (Qual.Pers), Quality of reception (Qual.Accu), Cost ;
- Three decision-makers : Decision-maker 1 (D1), Decision-maker 2 (D2), Decision-maker 3 (D3).

The decision-makers' matrices are given below :

Criteria →	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
Actions ↓ \ Weight →	6	3	2	4	3
Center 1	6	5	2	4	5
Center 2	5	6	3	3	4
Center 3	7	5	4	6	3
Center 4	6	4	5	3	6

Table 5.1 – Evaluation matrix and weight of decision-maker 1 D_1

Criteria →	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
Actions ↓ \ Weight →	7	5	3	3	4
Center 1	7	6	2	3	3
Center 2	6	5	2	5	3
Center 3	5	7	3	6	4
Center 4	5	4	4	4	3

TABLE 5.2 – Evaluation matrix and weight of decision-maker 2 D_2

Criteria →	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
Actions ↓ \ Weight →	6	4	2	3	3
Center 1	6	5	2	4	4
Center 2	7	6	3	5	3
Center 3	6	5	4	3	5
Center 4	5	4	3	6	4

TABLE 5.3 – Evaluation matrix and weight of decision-maker 3 D_3

5.2 Resolution of problem

Step 1 :Determination of the global weights of the criteria
According to tables (5.1), (5.2) and (5.3), we obtain the weight matrix W presented by relation (5.1).

$$W = \begin{array}{c} \\ w_j^1 \\ w_j^2 \\ w_j^3 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{pmatrix} 6 & 3 & 2 & 4 & 3 \\ 7 & 5 & 3 & 3 & 4 \\ 6 & 4 & 2 & 3 & 3 \end{pmatrix} \end{array} \tag{5.1}$$

According to equation (2.2), the global weights of the criteria are easily calculated. Let's consider a small $k = 0.1$ in order to reduce the influence of extreme values.

$$\begin{aligned} w_1 &= B_{0.1}\,(w_1^1, w_1^2, w_1^3) \\ &= B_{0.1}(6,7,6) \\ &= 0.1 * \log\left(\frac{\exp(\frac{6}{0.1}) + \exp(\frac{7}{0.1}) + \exp(\frac{6}{2})}{3}\right) \\ &= 6,89 \end{aligned}$$

$$\begin{aligned} w_2 &= B_{0.1}\,(w_2^1, w_2^2, w_2^3) \\ &= B_{0.1}(3,5,4) \\ &= 0.1 * \log\left(\frac{\exp(\frac{3}{0.1}) + \exp(\frac{5}{0.1}) + \exp(\frac{4}{2})}{3}\right) \\ &= 4,89 \end{aligned}$$

In a similar manner, the other global weights are obtained, thus forming the following matrix of global weights :

$$w = \begin{pmatrix} 6,890 & 4,890 & 2,890 & 3,890 & 3,890 \end{pmatrix}$$

Step 2 :Determination of the global evaluations of the alternatives
In this section, we have aggregated the performances of the alternatives based on the WOWA operator given by relation (2.1). We choose a linear function $\varphi(x) = x$ and an optimistic decision-making behavior (i.e., prioritizing good performances).

Example of calculating the performances according to the criterion c_1 (Equi.Resp) :

— **Weight vector**

In this section, we first extract the weight vector according to criterion c_1, denoted p^{c_1}, then normalize it, and finally reorder it according to the decision-making behavior. We obtain :

$$p^{c_1} = \begin{pmatrix} 6 & 7 & 6 \end{pmatrix}$$

By normalizing and reordering the components, we obtain :

$$p^{c_1}_{\sigma(k)} = \begin{pmatrix} 0.316 & 0.316 & 0.368 \end{pmatrix}$$

Thus, we have : : $\sum_{k=1}^{3} p^{c_1}_{\sigma(k)} = 1$ $\quad\sum_{k=2}^{3} p^{c_1}_{\sigma(k)} = 0.684$ $\quad\sum_{k=3}^{3} p^{c_1}_{\sigma(k)} = 0.368$

— **Global performances of the alternatives according to the criterion.** c_1 **(Equi.Resp)**
According to tables (5.1), (5.2), and (5.3), the performances of the **centers** provided respectively by decision-makers D_1, D_2, and D_3 according to criterion c_1 are represented by relation (5.2) :

$$\begin{pmatrix} & d_1 & d_2 & d_3 \\ Center1 & 6 & 7 & 6 \\ Center2 & 5 & 6 & 7 \\ Center3 & 7 & 5 & 6 \\ Center4 & 6 & 5 & 5 \end{pmatrix} \tag{5.2}$$

Given that g_{ij} is the global performance of alternative i with respect to criterion j, it is calculated by applying the formula given by (2.1) :

$$\begin{aligned} g_{11} &= \text{WOWA}(6,7,6) \\ &= (6-0)\times\varphi\left(\sum_{k=1}^{3} p^{c_1}_{\sigma(k)}\right) + (6-6)\times\varphi\left(\sum_{k=2}^{3} p^{c_1}_{\sigma(k)}\right) + (7-6)\times\varphi\left(\sum_{k=3}^{3} p^{c_1}_{\sigma(k)}\right) \\ &= 6\times\varphi(1) + 0\times\varphi(0.684) + 1\times\varphi(0.368) \\ &= 6 + 0.368 \\ &= 6.368 \end{aligned}$$

$$\begin{aligned} g_{21} &= \text{WOWA}(5,6,7) \\ &= (5-0)\times\varphi(1) + (6-5)\times\varphi(0.684) + (7-6)\times\varphi(0.368) \\ &= 5 + 0.684 + 0.368 \\ &= 6.052 \end{aligned}$$

In a similar manner, we calculate the global performances with respect to the other criteria. This gives us table (5.4) :

Criteria	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
Center 1	6,368	5,417	2,000	3,700	4,100
Center 2	6,053	5,750	2,714	4,400	3,400
Center 3	6,053	5,833	3,714	5,100	4,100
Center 4	5,368	4,000	4,143	4,500	4,500

TABLE 5.4 – Matrix of global performances

Step 3 :Formation of the global preference matrix
In this step, we form the global preference matrix based on w and the matrix of global performances. This gives us the table (5.5) below :

Criteria	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
weight	6,890	4,890	2,890	3,890	3,890
Center 1	6,368	5,417	2,000	3,700	4,100
Center 2	6,053	5,750	2,714	4,400	3,400
Center 3	6,053	5,833	3,714	5,100	4,100
Center 4	5,368	4,000	4,143	4,500	4,500

TABLE 5.5 – Global preference matrix

Step 4 :Application of SMART to the global preference matrix

— **Normalization of weights and calculation of utility values**
We calculate the normalized weights based on relation (**??**). In this problem, we have 5 criteria, of which 4 are beneficial and 1 is non-beneficial. This gives us the table (5.6) below :

Critères	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Cost
normalized weight	0,307	0,218	0,129	0,173	0,173
Center 1	1,000	0,773	0,000	0,000	0,364
Center 2	0,684	0,955	0,333	0,500	1,000
Center 3	0,684	1,000	0,800	1,000	0,364
Center 4	0,000	0,000	1,000	0,571	0,000

Table 5.6 – Matrix of normalized weights and utility values

— **Calculation and aggregation of weighted scores**
In this section, we calculate the weighted score $\widehat{u}_j(a_i)$ for each center i with respect to criterion j, and then aggregate the weighted scores of each center according to relation (2.5). This gives us the table (5.7) below :

	Center 1	Center 2	Center 3	Center 4
Sum of scores	0,538	0,721	0,767	0,228

Table 5.7 – Matrix of aggregated weighted scores

— **Selection of the best alternative**
In this final step, which is the selection of the best center, we choose the center with the highest score according to equation (2.6). It turns out that Center 3 has the highest score, making it the best center for handling severe COVID-19 cases. The ranking is provided in table (5.8) below :

	Scores	**Rank**
Center 1	0,538	3^{e}
Center 2	0,721	2^{e}
Center 3	0,767	1er
Center 4	0,228	4^{e}

Table 5.8 – Table of scores and rankings

6 Conclusions

This study demonstrates the value of extending the SMART method to group decision-making through the use of advanced aggregation operators such as WOWA and the weighted logarithmic exponential mean. This method introduces increased flexibility in the aggregation of preferences, allowing for precise control over priorities, particularly regarding good or poor performances. This approach not only promotes more harmonious decisions within groups but also strengthens social cohesion by taking into account differences in opinion and influence. MAC-SMART proves to be particularly useful for organizations looking to enhance their collective decision-making processes in increasingly complex contexts, offering a robust and flexible method suited to the diverse challenges of collaborative governance. This approach could be expanded to meet the growing needs of diverse organizations, optimizing collective decisions across a wide range of contexts.

7 Limitations

The limitations of this method include the complexity of implementing the WOWA operators and the weighted logarithmic exponential mean, which requires specific technical expertise, as well as the subjective setting of weights, which can influence the results. Additionally, MAC-SMART requires further empirical validations to ensure its effectiveness.

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