# Enhancing Decision-Making: Integrating Spherical Fuzzy-AHP with OPARA Method

*Saroj Koul [1#]; Ajaygopal KV [2]; Rakesh Verma [3]*

## Highlights

- *Approach:* Innovative as SF-OPARA integrates OPARA with SFS for complex scenarios, capturing all aspects in one framework.
- *Reliability*: SF-OPARA exhibits superior dependability to traditional methods, as demonstrated in the case study.
- *Applicability:* The handling of subjective data by SF-OPARA enhances broader decision-making and precision.

**ABSTRACT**

This study introduces "Spherical Fuzzy-Objective Pairwise Ratio Analysis (SF-OPARA)", a novel decision-making method that integrates OPARA with Spherical Fuzzy Sets (SFS) to address complex and uncertain scenarios. SF-OPARA overcomes the limitations of traditional methods that struggle with ambiguous data by combining OPARA's objective structure with the flexibility of SFS. This integration captures membership, non-membership, and hesitancy within a single framework, enhancing effectiveness in uncertain conditions. A case study demonstrates SF-OPARA's superior reliability to traditional and fuzzy Multi-criteria Decision-Making (MCDM) methods, providing detailed and practical rankings under uncertainty. SF-OPARA aligns decision-making with real-world needs by converting expert judgments into precise, actionable insights. Its unique handling of subjective data makes it a valuable tool across various fields, including public policy, resource management, and strategic planning. This approach enhances decision-making flexibility and precision, aiding organizations in making well-informed choices that reflect real-world complexities. SF-OPARA's broad applicability significantly adds to decision support tools, especially in uncertain environments. This study is the first to apply Spherical Fuzzy Sets within the OPARA framework, filling a notable gap in MCDM research and advancing the development of tools for complex decision challenges.

**Keywords:** Spherical Fuzzy Sets (SFS), MCDM, Objective Pairwise Ratio Analysis (OPARA), Uncertainty Handling, Decision Support Systems, Fuzzy Logic.

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## 1. Introduction

*Multi-criteria decision-making* (MCDM) methods are crucial for evaluating and ranking alternatives across complex criteria in various fields, including resource allocation, personnel selection, and location analysis **(Wind & Saaty, 1980).** Widely adopted MCDM methods, such as the *Analytic Hierarchy Process* (AHP) and *Technique for Order Preference by Similarity to Ideal Solution* (TOPSIS), rely on pairwise comparisons and ratio analysis, which may struggle to handle the uncertainty and ambiguity present in real-world data **(Van Laarhoven & Pedrycz, 1983; Kahraman et al., 2014)**. A more modern addition to MCDM, the Objective Pairwise Ratio Analysis (OPARA) method provides strong results in objective settings. However, unlike other older methods, its effectiveness in many real-world decision-making situations is limited due to its inability to completely handle uncertain or vague data (**Kutlu Gündodu & Kahraman, 2019**).

Fuzzy set theory and its extensions have been integrated into MCDM to address limitations in capturing uncertain and imprecise judgments **(Zadeh, 1965; Atanassov, 1986; Yager, 2013)**. Extensions like Intuitionistic Fuzzy Sets (IFS), Pythagorean Fuzzy Sets (PFS), and Neutrosophic Sets (NS) allow decision-makers to define degrees of membership, non-membership, and hesitation, enhancing MCDM's robustness (**Liu et al., 2017; Smarandache, 2003).** However, these sets do not independently handle each parameter, limiting flexibility. Spherical Fuzzy Sets (SFS), developed by **Kutlu Gündoğdu & Kahraman (2019),** overcome this by allowing independent adjustments for membership, non-membership, and hesitancy, providing a more adaptable tool for complex judgments **(Kutlu Gündoğdu & Kahraman, 2019).** This three-dimensional extension has shown promising results in MCDM applications like SF-AHP and SF-TOPSIS **(Gim & Kim, 2014; Rezaei et al., 2014).**

The SF-AHP offers advanced uncertainty modelling by using SFS with three dimensions (membership, non-membership, and hesitancy), allowing for a comprehensive representation of decision-maker uncertainty and independent hesitancy expression **(Jawad et al., 2024; Sharaf, 2021).** Converting fuzzy matrices to crisp matrices for Saaty's eigenvalue method enhances its consistency and reliability, achieving better consistency ratios than methods like fuzzy-BWM **(Sharaf, 2021; Haseli et al., 2024).** SFAHP also introduces automatic algorithms to reduce computational efforts, making it more efficient than traditional fuzzy-AHP methods **(Jawad et al., 2024; Kinay & Tezel, 2022).** Additionally, it is highly versatile, being successfully applied to fields like portfolio and supplier selection, with results comparable to or better than other fuzzy methods for complex matrices **(Jawad et al., 2024; Sharaf, 2021; Oztaysi et al., 2023).**

Despite OPARA's efforts to provide an objective comparison framework, it has not yet been extended to handle uncertain environments as effectively as newer fuzzy MCDM methods **(Kutlu Gündodu & Kahraman, 2020).** To address this gap, we propose the Spherical Fuzzy OPARA (SF-OPARA) method, combining OPARA's ratio-based approach with SFS to enable robust decision-making under uncertainty. This integration leverages OPARA's objective analysis while introducing spherical fuzzy flexibility to enhance performance in ambiguous scenarios. As such, this paper presents SF-OPARA as an innovative MCDM technique, highlighting its theoretical foundation, steps, and application in a case study.

## 2. Literature Review

MCDM techniques have evolved significantly to address the complexities of real-world decisions. While classical approaches such as AHP and TOPSIS have been widely adopted for applications requiring comparative evaluations across multiple criteria **(Wind & Saaty, 1980; Hwang et al., 1981**), the traditional MCDM methods operate under deterministic conditions, often limiting their effectiveness when faced with uncertain or vague data **(Chen, 2000; Van Laarhoven** & **Pedrycz, 1983).** As a result, extensions incorporating fuzzy set theory have emerged as valuable tools for handling imprecision in decision-making scenarios **(Zadeh, 1965; Bellman & Zadeh, 1970).**

***Spherical fuzzy sets: preliminaries***

The Intuitionistic and Pythagorean fuzzy membership functions are composed of membership, non-membership, and hesitancy parameters and are calculated by $\pi_{\tilde{I}} = 1 - \mu_{\tilde{I}} - v_{\tilde{I}}$ or $\pi_{\tilde{p}} = \left(1 - \mu_{\tilde{p}}^2(u) - v_{\tilde{p}}^2(u)\right)^{1/2}$, respectively. Neutrosophic membership functions are also defined by three parameters: truthiness, falsity, and indeterminacy, whose sum can be between 0 and 3, and the value of each is between 0 and 1 independently. In spherical fuzzy sets, while the squared sum of membership, non-membership, and hesitancy parameters can be between 0 and 1, each of them can be defined between 0 and 1 independently to satisfy that their squared sum is at least equal to 1 **(Gül, 2021).**

**Figure 1** illustrates the differences among IFS, PFS, NS, and SFS. We now define SFS and summarize spherical distance measurement, arithmetic operations, aggregation operators, and defuzzification operations.

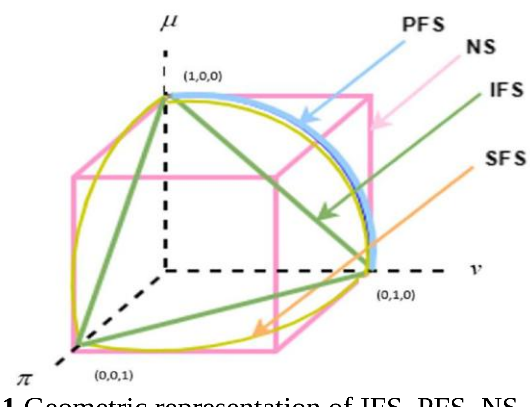


**Figure. 1** Geometric representation of IFS, PFS, NS, and SFS.

**Definition 1** (Spherical fuzzy sets (SFS) $\tilde{\boldsymbol{A}}_S$ ) Let $U_1$ and $U_2$ be two universes **(Mehdi et al., 2024)**. Let two spherical fuzzy sets $\tilde{A}_S$ and $\tilde{B}_S$ of the universe of discourse $U_1$ and $U_2$ be as follows:

$$\tilde{A}_S = \left\{x, \left(\mu_{\tilde{A}_S}(x), v_{\tilde{A}_S}(x), \pi_{\tilde{A}_S}(x)\right) \mid x \in U_1\right\} \quad (1)$$

where

$\mu_{\tilde{A}_S}(x): U_1 \to [0,1]$, $v_{\tilde{A}_S}(x): U_1 \to [0,1]$,

$\pi_{\tilde{A}_S}(x): U_1 \to [0,1]$ and $0 \leq \mu_{\tilde{A}_S}^2(x) + v_{\tilde{A}_S}^2(x) + \pi_{\tilde{A}_S}^2(x)1\ \forall x \in U_1$

For each $x$, the $\mu_{\tilde{A}_S}(x), v_{\tilde{A}_S}(x)$ and $\pi_{\tilde{A}_S}(x)$ are the degrees of membership, non-membership, and hesitancy of $x$ to $\tilde{A}_S$, respectively.

$$\tilde{B}_S = \{y, (\mu_{\tilde{B}_S}(y), v_{\tilde{B}_S}(y), \pi_{\tilde{B}_S}(y)) \mid y \in U_2\} \quad (2)$$

where

$\mu_{\tilde{B}_S}(y): U_2 \rightarrow [0,1]$, $v_{\tilde{B}_S}(y): U_2 \rightarrow [0,1]$,

$\pi_{\tilde{B}_S}(y): U_2 \rightarrow [0,1]$ and $0 \leq \mu^2_{\tilde{B}_S}(y) + v^2_{\tilde{B}_S}(y) + \pi^2_{\tilde{B}_S}(y) \leq 1 \; \forall y \in U_2$

For each $y$, the numbers $\mu_{\tilde{B}_S}(y), v_{\tilde{B}_S}(y)$ and $\pi_{\tilde{B}_S}(y)$ are the degrees of membership, non-membership, and hesitancy of $y$ to $\tilde{B}_S$, respectively (**Kutlu Gündoğdu & Kahraman 2019a).** The mathematical operations of spherical fuzzy numbers are in **Annexure 1.**

## 3. Integrated SF-AHP-OPARA Methodology

The proposed method integrates SF-AHP for criteria weights and OPARA for alternative ranking. The proposed **SF-AHP-OPARA** method comprises several steps (**Annexure-2).**

The steps of the proposed method are as follows:

*STEP 1.* Construct a Hierarchical Structure.

*STEP 2.* Pairwise Comparisons Using Spherical Fuzzy Matrices: Create pairwise comparisons using linguistic terms and calculate score indices using predefined equations.

*STEP 3.* Consistency Check of Matrices: Convert linguistic terms to numerical values and apply consistency tests. Ensure the *Consistency Ratio* (CR) is below 10%.

*STEP 4.* Calculate Spherical Fuzzy Local Weights: Compute weights of criteria and alternatives using the weighted arithmetic mean (SWAM operator).

*STEP 5.* Aggregate to Global Weights: Combine weights from all levels to estimate global preferences. Use either defuzzification methods or continue with fuzzy values.

*STEP 6.* Construct Decision Matrix: Define the *decision matrix* with criteria weights. Ensure all values are positive.

*STEP 7.* Compute *Range-Based Pairwise Adjusted Ratios* (RPAR): Use the RPAR formula to adjust for the benefit and cost criteria using adjustment parameters ρ.

*STEP 8.* Calculate *Linearity-Based Pairwise Adjusted Ratios* (LPAR): Compute LPAR for non-linear effects of criteria based on τ, the linearity adjustment parameter.

*STEP 9.* Aggregate *Pairwise Adjusted Ratios* (APAR): Combine RPAR and LPAR using a weighted aggregation parameter ω.

*STEP 10.* Determine Final Scores and Rank Alternatives: Calculate the final score for each alternative using aggregated pairwise ratios. Rank alternatives based on their scores.

The CR (STEP 3) applies to the SF-AHP because it ensures the reliability and consistency of the pairwise comparison matrices used in decision-making. Also, CR is crucial for maintaining the integrity of the decision-making process, as it identifies and corrects any inconsistencies in the pairwise comparisons (**Kutlu Gündodu & Kalman, 2020**).

## 4. Numerical Case Study

In India, Ladakh's mega solar power project is a significant initiative to harness the region's abundant solar energy potential to contribute to India's renewable energy goals **(Hindustan Times, 2023)**. The project involves the development of a 10 GW solar power plant and merging with India's ambitious target of producing 500 GW of renewable energy by 2030 **(Times of India, 2023; Ladakh Energy, 2024)**.

The primary regions leveraging the Ladakh area's high altitude and abundant sunshine and planning for solar development include Pang, Leh, Changthang, and Kargil. These four regions have been chosen to maximize the efficiency and impact of solar energy projects, contributing to Ladakh's sustainable energy future. For this numerical case, we shall study these four locations ***(Pang, Leh, Changthang, and Kargil).***

After a comprehensive literature review, **Kutlu Gündoğdu & Kahraman (2020)** determined four criteria and 12 sub-criteria. Criteria are environmental conditions (C1), economic situations (C2), technological opportunities (C3), and site characteristics (C4). **Figure 2** illustrates this hierarchy, which consists of all criteria and sub-criteria related to them. In this structure, while "economic situations" are a non-beneficial criterion, the rest are beneficial. First of all, the assessments for the criteria and sub-criteria are collected from a decision-makers group for the goal, using the linguistic terms given in **Table 1.**

**Table 1**: Linguistic Terms

| | $(\mu, v, \pi)$ | Score Index (SI) |
|---|---|---|
| Absolutely more importance (AMI) | (0.9,0.1,0.0) | 9 |
| Very high importance (VHI) | (0.8,0.2,0.1) | 7 |
| High importance (HI) | (0.7,0.3,0.2) | 5 |
| Slightly more importance (SMI) | (0.6,0.4,0.3) | 3 |
| Equally important (EI) | (0.5,0.4,0.4) | 1 |
| Slightly low importance (SLI) | (0.4,0.6,0.3) | 1/3 |
| Low importance (LI) | (0.3,0.7,0.2) | 1/5 |
| Very low importance (VLI) | (0.2,0.8,0.1) | 1/7 |
| Absolutely low importance (ALI) | (0.1,0.9,0.0) | 1/9 |

**Table 2:** Comparison Matrix

| Criteria | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| C1 | EI | HI | AMI | SMI |
| C2 | LI | EI | SMI | LI |
| C3 | ALI | SLI | EI | VLI |
| C4 | SLI | HI | VHI | EI |

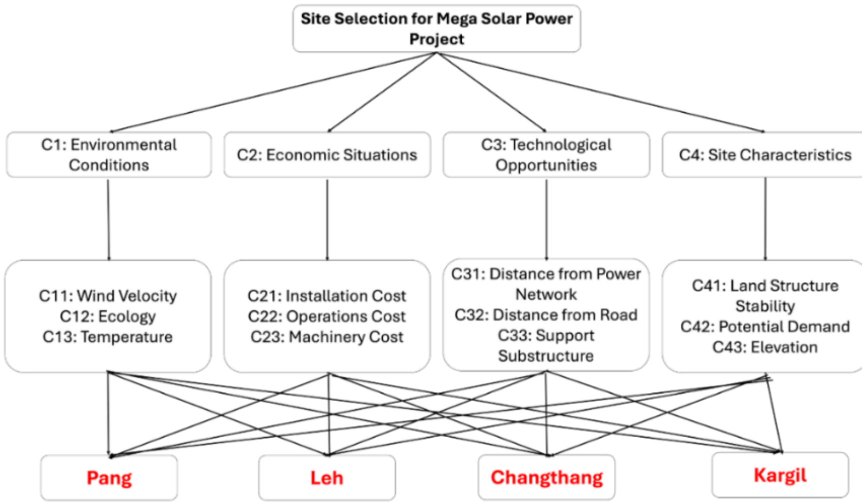


**Figure 2:** Hierarchical structure for the problem

For this study, we have limited our scope to independent criteria for evaluation. In the future, with more data available, interdependencies shall be examined.

## 5. Results/Model Analysis

The results of the comparative analysis of alternative rankings using AHP, SF-AHP, OPARA, and the proposed SF-AHP-OPARA method are presented in **Table 3.**

The objective of this comparison is to evaluate the consistency and effectiveness of the proposed SF-AHP-OPARA method with traditional methods. The rankings of the alternatives (**A1, A2, and A3**) show consistent patterns across several methods, particularly between the proposed SF-AHP-OPARA and the standard AHP and SF-AHP methods. Specifically, Alternatives **A2 and A3** are ranked identically across the AHP, SF-AHP, and SF-AHP-OPARA methods, securing second and third places, respectively. This consistency suggests a high degree of alignment in prioritizing alternatives when considering subjective and objective aspects through these methods. In contrast, the OPARA method diverges slightly by ranking **A1 and A3** differently. According to OPARA, Alternative **A1** ranks third, while Alternative **A3** is placed in second position. This variation may indicate that OPARA weighs specific criteria differently than the fuzzy-enhanced AHP methods, possibly due to differences in the underlying principles or assumptions within the OPARA methodology.

**Table 3:** Comparison of Alternative Rankings

| ***Alternatives*** | ***AHP*** | ***SF-AHP*** | ***OPARA*** | ***SF-AHP-OPARA*** |
|---|---|---|---|---|
| ***A1*** | 1 | 1 | 3 | 3 |
| ***A2*** | 2 | 2 | 1 | 1 |
| ***A3*** | 3 | 3 | 2 | 2 |

The proposed SF-AHP-OPARA method aligns closely with both AHP and SF-AHP, maintaining the original AHP-based rankings for **A1, A2, and A3** while integrating the robustness of the OPARA methodology. Thus, SF-AHP-OPARA successfully retains consistency with traditional AHP-based methods and provides additional depth by incorporating elements from OPARA. This outcome supports the validity of SF-AHP-OPARA as a hybrid method that synthesizes the strengths of both AHP and OPARA approaches while ensuring stable ranking outputs.

In summary, the comparative analysis illustrates that SF-AHP-OPARA achieves a balanced approach, integrating both subjective judgments and objective assessments effectively. The consistency in ranking across methods for **A2 and A3** and the minor discrepancy observed with OPARA for **A1** indicate that SF-AHP-OPARA offers a reliable alternative for multi-criteria decision-making contexts where both fuzzy logic and objective analysis are pertinent.

## 6. Conclusions

This study introduced the SF-AHP-OPARA method as a novel and robust approach for multi-criteria decision-making under uncertainty, combining the objective analytical strengths of OPARA with the adaptive capabilities of Spherical Fuzzy Sets. By integrating both objectives, pairwise comparisons and the nuanced representation of ambiguity through membership, non-membership, and hesitancy values, SF-AHP-OPARA offers a significant advancement in decision-making methodologies where traditional approaches fall short. The comparative analysis demonstrated that SF-AHP-OPARA maintains consistent rankings with traditional AHP-based methods while providing additional depth by comprehensively addressing uncertainty.

The case study results further validate SF-AHP-OPARA's applicability and effectiveness, particularly in incomplete or ambiguous information situations. The consistency observed in ranking alternatives highlights SF-AHP-OPARA's alignment with established MCDM approaches while preserving the flexibility needed to handle subjective judgments effectively. The observed alignment with AHP and SF-AHP rankings, alongside the minor discrepancies with OPARA, underscores the method's capability to balance subjective and objective elements in decision-making, making it a reliable alternative to existing methods. SF-AHP-OPARA's potential for application across diverse fields, such as public policy, resource management, and strategic planning, is notable. This method facilitates well-informed decisions that reflect the inherent complexities of real-world scenarios by providing decision-makers with a more precise and flexible tool. In conclusion, SF-AHP-OPARA represents a valuable contribution to the field of MCDM, bridging a critical gap in handling objective and subjective data. Future research could explore its applications across different sectors to establish its utility further and refine its adaptability to various decision-making environments.

Given the reality of our physical world, no study is perfect. The data related to each site was collected from secondary sources of data. More precise primary data can reveal interesting changes in the site location. Although spherical fuzzification of criteria weights allows the experts to express their preferences, the objective valuation similar to the OPARA method can be used to determine the criteria weights in the future.

## 7. Key References


Amiri, M., Hashemi-Tabatabaei, M., Keshavarz-Ghorabaee, M., Kaklauskas, A., Zavadskas, E. K., & Antucheviciene, J. (2022). A fuzzy extension of the simplified best-worst method (F-SBWM) and its applications to decision-making problems. *Symmetry*, *15*(1), 81.

Atanassov, A.I. (1986). Sugar beet (Beta vulgaris L.). In *Crops I* (pp. 462-470). Berlin, Heidelberg: Springer Berlin Heidelberg.

Bellman, R.E., & Zadeh, L.A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141.

Chen, C.T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy sets and systems*, *114*(1), 1-9.

Gim, B., & Kim, J.W. (2014). Multi-criteria evaluation of hydrogen storage systems for automobiles in Korea using the fuzzy analytic hierarchy process. *International Journal of Hydrogen Energy*, *39*(15), 7852-7858.

Gül, S. (2021). Spherical fuzzy version of EDAS and an application. *International Journal of Advances in Engineering and Pure Sciences*, *33*(3), 376-389.

Haseli, G., Sheikh, R., Ghoushchi, S. J., Hajiaghaei-Keshteli, M., Moslem, S., Deveci, M., & Kadry, S. (2024). An extension of the best–worst method based on the spherical fuzzy sets for multi-criteria decision-making. Granular Computing, 9(2), 40.

Hindustan Times. (2023). Ladakh's mega solar power project delayed, to be ready by 2026. Retrieved Nov-3, 2024, from https://www.hindustantimes.com/cities/chandigarh-news/ladakhs-mega-solar-power-project-delayed-to-be-ready-by-2026-101675761954786.html

Hwang, C. L., Yoon, K., Hwang, C. L., & Yoon, K. (1981). Methods for multiple attribute decision making. *Multiple attribute decision making: methods and applications a state-of-the-art survey*, 58-191.

Jawad, M., Naz, M., & Muqaddus, H. (2024). A multi-criteria decision-making approach for portfolio selection by using an automatic spherical fuzzy AHP algorithm. Journal of the Operational Research Society, 75(1), 85-98.

Kahraman, C., Öztayşi, B., Sarı, İ. U., & Turanoğlu, E. (2014). Fuzzy analytic hierarchy process with interval type-2 fuzzy sets. *Knowledge-Based Systems*, *59*, 48-57.

Kinay, A.O., & Tezel, B.T. (2022). Modification of the fuzzy analytic hierarchy process via different ranking methods. Int'l Journal of Intelligent Systems, 37(1), 336-364.

Kutlu Gündoğdu, F., & Kahraman, C. (2019a). A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection. *Journal of Intelligent & Fuzzy Systems*, *37*(1), 1197-1211.

Kutlu Gündoğdu, F., & Kahraman, C. (2019b). A novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets. *Engineering Applications of Artificial Intelligence*, *85*, 307-323.

Kutlu Gündoğdu, F., & Kahraman, C. (2019c). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of intelligent & fuzzy systems*, *36*(1), 337-352.

Kutlu Gundogdu, F., & Kahraman, C. (2019d). Extension of WASPAS with spherical fuzzy sets. *Informatica*, *30*(2), 269-292.

Kutlu Gündoğdu, F., & Kahraman, C. (2020). A novel spherical fuzzy analytic hierarchy process and its renewable energy application. *Soft Computing*, *24*, 4607-4621.

Ladakh Energy. (2024). Ladakh Energy. Retrieved Nov-3, 2024, from http://ladakhenergy.org/

Liu, F., Peng, Y., Zhang, W., & Pedrycz, W. (2017). On consistency in AHP and fuzzy AHP. *Journal of Systems Science and Information*, *5*(2), 128-147.
Mehdi, K.G., Abdolghani, R., Maghsoud, A., Zavadskas, E.K., & Antuchevičienė, J. (2024). Multi-Criteria personnel evaluation and selection using an objective pairwise adjusted ratio analysis (OPARA). *Economic computation and economic cybernetics studies and research.*, *58*(2), 23-45.
Oztaysi, B., Onar, S. C., & Kahraman, C. (2023). An Initial Research on Comparison of Fuzzy AHP and Classical AHP Methods. In International Conference on Intelligent and Fuzzy Systems (pp. 382-388). Cham: Springer Nature Switzerland.
Rezaei, S., Amin, M., & Ismail, W.K.W. (2014). Online repatronage intention: an empirical study among Malaysian experienced online shoppers. *International Journal of Retail & Distribution Management*, *42*(5), 390-421.
Saaty, T.L., & Peniwati, K. (2013). *Group decision making: drawing out and reconciling differences*. RWS publications.
Saaty, T.L., & Shang, J.S. (2007). Group decision-making: Headcount versus intensity of preference. *Socio-Economic Planning Sciences*, *41*(1), 22-37.
Sharaf, I.M. (2021). Global supplier selection with spherical fuzzy analytic hierarchy process. Decision making with spherical fuzzy sets: Theory and applications, 323-348.
Smarandache, F. (2003). *Definitions derived from neutrosophics*. Infinite Study.
Times of India. (2023). Government clears ₹20,000 crore power line to energize showpiece Ladakh solar project. Retrieved Nov-3, 2024, from https://timesofindia.indiatimes.com /business/india-business/government-clears-rs-20000-crore-power-line-to-energise-showpiece-ladakh-solar-project/articleshow/104534160.cms
Van Laarhoven, P.J., & Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems*, 11(1-3), 229-241.
Wind, Y., & Saaty, T.L. (1980). Marketing applications of the analytic hierarchy process. *Management Science*, 26(7), 641-658.
Yager, R.R. (2013). Pythagorean membership grades in multi-criteria decision making. *IEEE Transactions on Fuzzy Systems*, 22(4), 958-965.
Zadeh, L.A. (1965). Fuzzy sets. Information and Control.

**Annexure 1:** Mathematical Operations of Spherical Fuzzy Numbers

| Operation | Definition |
|---|---|
| Addition of Spherical Numbers | $\tilde{A}_S \oplus \tilde{B}_S = \Big\{z, \Big(\max_{z=x+y} \min\{\mu_{\tilde{A}_S}(x), \mu_{\tilde{B}_S}(y)\}\Big)$ $\Big(\min_{z=x+y} \max\{v_{\tilde{A}_S}(x), v_{\tilde{B}_S}(y)\}\Big), \Big(\min_{z=x+y} \min\{\pi_{\tilde{A}_S}(x), \pi_{\tilde{B}_S}(y)\}\Big)\Big\}$ |
| Multiplication of Spherical Numbers | $\tilde{A}_S \otimes \tilde{B}_S = \{z, (\max_{z=x*y} \min\{\mu_{\tilde{A}_S}(x), \mu_{\tilde{B}_S}(y)\}),$ $(\min_{z=x*y} \max\{v_{\tilde{A}_S}(x), v_{\tilde{B}_S}(y)\}), (\min_{z=x*y} \min\{\pi_{\tilde{A}_S}(x), \pi_{\tilde{B}_S}(y)\})\}$ |
| Union of Spherical Numbers | $\tilde{A}_S \cup \tilde{B}_S = \{\max\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\}, \min\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\},$ $\min\Big\{\Big(1 - \big((\max\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\})^2 + (\min\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\})^2\big)\Big)^{1/2}, \max\{\pi_{\tilde{A}_S}, \pi_{\tilde{B}_S}\}\Big\}\Big\}$ |
| The intersection of Spherical Numbers | $\tilde{A}_S \cap \tilde{B}_S = \{\min\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\}, \max\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\},$ $\max\Big\{\Big(1 - \big((\min\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\})^2 + (\max\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\})^2\big)\Big)^{1/2}, \min\{\pi_{\tilde{A}_S}, \pi_{\tilde{B}_S}\}\Big\}\Big\}$ |
| Multiplication of Spherical Number by Scalar | $\lambda \cdot \tilde{A}_S = \Big\{\Big(1 - (1 - \mu_{\tilde{A}_S}^2)^\lambda\Big)^{1/2}, v_{\tilde{A}_S}^\lambda, \Big((1 - \mu_{\tilde{A}_S}^2)^\lambda - (1 - \mu_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2)^\lambda\Big)^{1/2}\Big\}$ |
| Spherical Numbers raised by a Scalar | $\tilde{A}_S^\lambda = \Big\{\mu_{\tilde{A}_S}^\lambda, \Big(1 - (1 - v_{\tilde{A}_S}^2)^\lambda\Big)^{1/2}, \Big((1 - v_{\tilde{A}_S}^2)^\lambda - (1 - v_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2)^\lambda\Big)^{1/2}\Big\}$ |
| Spherical Weighted Arithmetic Mean (SWAM) | $\mathrm{SWAM}_w(\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn})$ $= w_1\tilde{A}_{S1} + w_2\tilde{A}_{S2} + \cdots + w_n\tilde{A}_{Sn}$ $= \Big\{\Big[1 - \prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2)^{w_i}\Big]^{1/2}, \prod_{i=1}^{n} v_{\tilde{A}_{Si}}^{w_i},$ $\Big[\prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2)^{w_i} - \prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2 - \pi_{\tilde{A}_{Si}}^2)^{w_i}\Big]^{1/2}\Big\}$ |

## **Annexure 2:** Proposed Integrated Method